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Linear Gyro-Resonance as a Cause of 2-4 Hz Sidebands Observed at Roberval, Canada

Anil Kumar[1*], D.P. Singh[2], Mukesh Kumar[3], Narvinder Singh[4], Krishan Kumar[5], Ashwani Kumar[6] and Manisha Kumari[7]

[1]Department of Physics, P.D.M. College of Engineering, Bahadurgarh, Jhajjar, Haryana-124507, India*
[2]P.T.I.M.T., Chaudharywas, Hisar, Haryana-125001, India
[3]Department of Physics, B.M. College of Management and Technology, Farrukhnagar, Gungaon, Haryana-122506, India
[4]Department of Physics, B.M. College of Management and Technology, Farrukhnagar, Gungaon, Haryana-122506, India
[5]Modern Public School, Bhiwadi, Rajasthan-122504, India
[6]Department of Physics, T.R.A. Memorial Institute of Engg. & Technology, Mandi, H.P.-175008, India
[7]Department of Physics, F.G.M. Government College, Adampur, Hisar, Haryana-125052, India
**E-mail: chhikara_anil05@rediffmail.com, chhikara.anil@gmail.com*

Abstract

A relation between 2-4 Hz side band generations with linear cyclotron resonance mechanism is studied taking into consideration the established theory of Inan (1977) that non-linear interaction needs minimum wave magnetic amplitude of 3-5 pT for wave-particle interaction to occur in the magnetosphere. It is shown that wave magnetic amplitude (Bw) only of ~2pTisrequired to produce 2-4 Hz side band spacing, which is below 3 pT indicating linear gyro-resonant interaction as a source to generate small side bands.

Keywords: Scattering of particles, Wave-particle interaction, Cyclotron Resonance.

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Introduction

Assuming both kinds of magnetic field- static over the time scales of interest, as well as dynamic, Gail et al(1990), Albert(2000) and Horne et al(2005)adopted experimental and theoretical methods to study Wave-Particle Interactions(WPI) in the magnetosphere. Applicability of quasi-linear (linear) and nonlinear amplification mechanisms was examined in both cases (Dowden, 1971, Dowden et al, 1978). Similar to production of x-rays, aurora in the magnetosphere, and D & E-region perturbation, side band generation is also a result of wave-particle. Dowden et al[1978] studied linear as well as non-linear amplification mechanisms using spectrograms/phasograms of 6. 6 kHz signals which were transmitted from Anchorage, Alaska, a transportable(TVLF) station comprising of VLF transmitter and balloon-lofted antenna being used. The transmitted signals were received, in this case, in the conjugate area located at Dunedin, New Zealand and Campbell Island. The experiment was done in August-September, 1973 and radiated power of transmitted signals was 93watt. On the basis of the analysis of linear and nonlinear amplification mechanisms Dowden et al(1978), Dowden(1971), Karpman et al(1974) and Koons et al(1976) differentiated them as under:

1. Linear amplification mechanism is most obvious when long trains of whistlers are observed, some times with increasing amplitude in successive hops. Mid-latitude hiss appears to be a consequence of amplification gained over several hops. Some amplification may be required in all observed whistlers. Measurement (McPherson et al, 1974a, b) using artificial signals indicate magnification of 25dB/hop. Sometimes fairly narrow band (`~1kHz), linear amplification is frequency dependent, and varies considerably over periods of several minutes. However, an essential feature is that the output/received signal is proportional to the input signal.
2. On the other hand, in nonlinear amplification the output amplitude is not related to the input amplitude provided that this is above some threshold, suggesting that the nonlinear amplification needs input wave's magnetic field amplitude (Bw) above some threshold. Inan (1977) and Inan et al (1978) found this limit to be between 3-5 pT. The output amplitude may be very large, but even when it is not; the distinguishing feature of non-linear amplification is that the output amplitude takes a finite time to grow. Sometimes it is the only obvious feature and appears as slowness in response to transmitter pulse or amplitude modulation (Dowden et al, 1978) as well as phase reversal modulation (Koons et al, 1976). In typical events the nonlinear nature is more obvious (Dowden, 1971, Dowden et al, 1978).

Side Band Observations

Bell and Helliwell(1971), Likhter et al(1971), Park and Chang(1978), Helliwell (1979), Matsumoto(1979) and Park(1981), have made observations of whistler-mode side band instability in the magnetosphere. Bell and Helliwell (1971) observed transmissions from NAA (located in Cutler, Maine) at 14. 7 and 17. 8 kHz when they analyzed the signal observed at Eights, Antarctica (75^0S, 77^0W), using a 300 Hz wide filter. Likhter et al(1971) reported observations of VLF signals with amplitude fluctuations with period of 0. 1-0. 5 sec from a Russian transmitter. Park and Chang(1978) and Chang et al(1980) showed examples of sideband generation recorded at Roberval[Quebec, Canada] (48^0N, 73^0W). These sideband were a result of VLF signals transmitted into the magnetosphere by the transmitters established at Siple, Antarctica(76^0S, 84^0W ; L= 4. 23, L is McIlwain parameter). It was observed generally that when a monochromatic wave was injected into the magnetosphere, the output wave often contained frequencies different from the transmitted frequency. Inan[1977], and Inan et al[1978] have shown that for non-linear gyro-resonant interaction between coherent signals and energetic electrons the wave magnetic amplitude (Bw) must be between ~5 pT.

Experimental observation of very small (2-4 Hz) side bands

VLF transmitter experiments conducted in July, 1977 at Siple, Antarctica(L~ 4) show that key-down signals sent into the magnetosphere often generate side bands as a result of wave-particle interactions between injected whistler mode VLF waves and resonant electrons. After a great analysis of spectral characteristics of recorded spectrograms, Park [1981] reported following properties of these side bands:

1. Side band amplitudes may be symmetrical/ asymmetrical about the carrier, but in asymmetrical case it is usually the upper side band that is stronger.
2. The side band frequency spacing varies from very small value of 2 Hz to 100 Hz(pl. see 8th line of abstract, p. 2286; 9th line of summary, p. 2289), but it bears no simple relationship to the carrier amplitude. In the same manner, side band amplitudes, too, have no relation with the injected signal/carrier frequency.
3. The side band amplitude is usually 10 dB or more below the carrier amplitude, but some time it can exceed the carrier amplitude and also trigger emissions Park[1981] explained side band spacing upto 20 -100 Hz (adopting non-linear amplification mechanism), no consideration was given to 2-4 Hz side band spacings. Here we try to show that these side bands separations can be explained using linear amplification mechanism.

Method of Computation

Ikeda (2002) tested the possibility of the side band generation in whistler mode adopting non-linear Doppler-shifted cyclotron interaction between energetic electrons and the whistler mode carrier signal. The energetic electrons resonate with the quasi-monochromatic whistler mode signal to generate sidebands as well as broadening of the transmitted carrier frequency. He derived following expression for side band spacing generated because of interaction between resonant electrons and transmitted/interacting whistler mode signal ($f_H > f$, f_H is cyclotron frequency of energetic electrons and f, the wave frequency).

$$F_{spacing} = 0.0633\, n.\ \sqrt{(k.\ V_\perp.\ \Omega(Bw))} \quad (1)$$

where n is order of side band generation, k is wave number, $V_\perp$ is perpendicular speed of resonant electrons and Ω(Bw) is the angular frequency of wave(trapping) magnetic field(also known as wave gyro frequency). Wave number k can be calculated from following formula (Singh, 1991, 1992).

$$kc=6.\ 28f\mu \quad (2)$$

where c is speed of light and μ is known as refractive index of the medium which can be computed from following equation(Singh, 1991, 1992).

$$\mu = \sqrt{\{f_p^2/f\,(f_H-f)\}} \quad (3)$$

f_p is plasma frequency of electrons. The resonant velocity of electrons (V_R) can be calculated from the equation that follows:

$$k.\ V_R = 6.\ 28(f_H - f) \quad (4)$$

Electron density to calculate angular plasma frequency at considered L shell of 4. 23 (where the duct between Siple and Roberval was found to be located at) was taken to be 313 el. cm^{-3}. This value corresponds to diffusive equilibrium model of Angerami and Thomas [1963] and have been used earlier by Singh & Singh [2006], and Singh et al[1994]. Energetic electrons gyro frequency f_H can be computed from following equation.

$$f_H \text{ (in kHz)} = 873.\ 6/L^3 \quad (5)$$

Results and discussion

The Siple experiment conducted at two interacting signal frequencies- 4020 and 4440 Hz. So we too, calculate our data at these two frequencies. Since threshold for non-linearity is ~5 pT, we compute our values at 4 pT in Table-1 for first order of side band spacing[see Eq. (1), n=1]. Eq. (1) clearly indicates that minimum spacing will be caused at low pitch angles but as only those pitch angles will contribute towards wave growth[Inan, 1977] for which $\alpha \geq \alpha_0$, we adopt 8^0- 20^0 pitch angles. Table-1 shows $F_{spacing}$ at the two considered frequency. It is evident from the Table-1 that as pitch angle increases side band spacing increases but decreases as frequency increases. It is clear from the table that we need Bw below 4 pT as all values of side band spacing are higher than 2 Hz. Though Inan(1977)] and Inan et al(1978) have shown that for non-linear interaction Bw should have a minimal of 5 pT but 4 pT can be considered as a threshold for non-linearity. Thus we compute side band spacing at 1pT-3pT at intervals of 0. 5pT and show these values in fig. 1. Fig. 1 shows that we may get 2-4 Hz spacing at both transmitter frequencies for Bw1-2pT indicating that 2-4 Hz spacing were perhaps due to linear amplification. It also suggests that that we can not get 2-4 Hz spacing at considered pitch angles of 8^0 -20^0, but may get it for Bw having values 1-2pT at pitch angles less than 8^0. Because of this, for computation of side band spacing, we consider{mean value of 1 and 2 pT} wave amplitude of 1. 5 pT and now consider minimum possible pitch angle to cause wave growth(i. e. loss cone pitch angle)α_0 i. e. half loss cone pitch angle. As already written Park [1981] has shown that the duct was located in this case at L=4. 23. At this location half loss cone pitch angle (Singh, 1991, 1992) is found to be 5. 03^0. Table 2 shows various data suggesting that 2-4 Hz side bands were generated by linear cyclotron mechanism, but to prove our point we study 2 tests for the purpose.

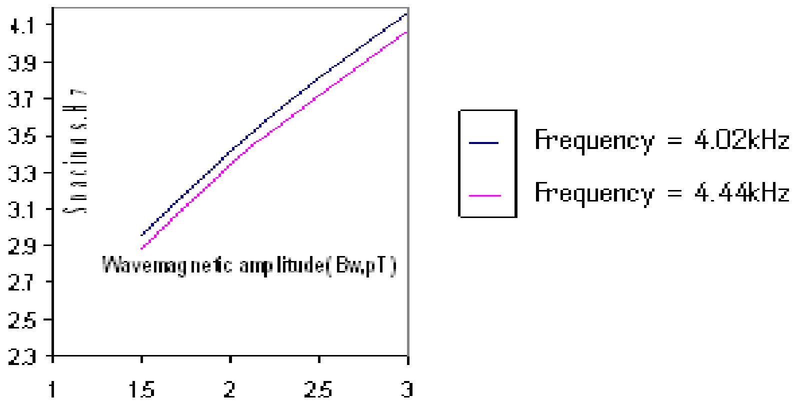

Figure 1: Variation of sideband spacings (Hz) with wave magnetic amplitude (Bw, in pT).

Table 1: $F_{spacing}$ (Hz) computed at three different pitch angles(8^0, $12^{0,}$ 16^0, 20^0) and Interacting signals of 4. 02 and 4. 44 kHz. The wave magnetic amplitude is taken to be 4 pT.

Pitch Angles	f =4020 Hz	f =4440Hz
8^0	4. 32	4. 20
12^0	5. 32	5. 17
16^0	6. 18	6. 00
20^0	6. 96	6. 76

Table 2: Various data used in the calculation of number of Trap Oscillation (N).

L = 4. 23	
n = 313electrons/cm^3	
f_H = 11540 Hz	
f = 4020 Hz	f = 4440 kHz
k = 2. 43(10^{-3}), m^{-1}k = 2. 63(10^{-3}), m^{-1}	
μ = 28. 89	μ = 28. 29
V_R = 1. 94(10^9), cm/s	V_R = 1. 69(10^9), cm/s
Ω(Bw) = 0. 2637 rad/s	Ω(Bw) = 0. 2637 rad/s
L_{int} = 45. 830 km	L_{int} = 49. 695 km
ω_T = 34. 64 rad/s	ω_T = 33. 64 rad/s

Test of non-linearity: Trap oscillations

Whether we are correct or not can only be checked using test of non-linearity. Inan(1977), and Inan et al(1978) have utilized this test to discuss non-linear wave-particle interactions between energetic electrons and whistler mode waves taking place in the magnetosphere considering the top of the 60^0 geomagnetic field line (L = 4) as the region for effective interaction. Cornilleau-Wehrlin and Gendrin(1979)[23] as well as Raghuram et al(1977)[24] have shown that to explain generation of side bands associated with received VLF signals at L=3. 8, the wave magnetic amplitude should be of the order of 4pT, and the phenomenon can not be explained by linear mechanism. In our case this value is below 4 pT and so linear mechanism may be taking place in case of events we are discussing here. But whereas these workers do their calculations at $\alpha = 45^0$, in our case this angle is equatorial half loss cone pitch angle ($\alpha \sim \alpha_0$). Because of this we take the help of the 'method of trap Oscillation' adopted earlier by Dowden (1971), Brinca (1972) and Inan(1977). Number of trap oscillations (N) in non-linear amplification should have a threshold of 2. As written earlier, Table-2 shows various data used in the calculation of number of trap oscillations, and other important parameters. We find that N can be computed for the data of Table-2 as under:

$$N =\omega_T.\ L_{int}/(6.\ 28V_R) \tag{6}$$

where $\omega_T$is trap oscillation frequency(in rad/s) and L_{int} is the trapping length which is a function of wave magnetic amplitude(Bw), pitch angle of the trapped electron(α), and resonant velocity(V_R). Trap oscillation frequency (ω_T) and trapping length (L_{int}) can be expressed as:

$$\omega_T = \sqrt{\{k.\ V_\perp.\ \Omega(Bw)\}} \tag{7}$$

$$L_{int}= 2.\ \Omega(Bw).\ (LR_E)^2.\ \tan\alpha.\ \{3/(1-x) +\tan^2\alpha\}/9.\ V_R \tag{8}$$

In Eq. (8), $R_E$is radius of earth(6370 km) and x is normalized frequency. We see that for the transmitted frequency of 4020 Hz, Bw =1. 5 pT and pitch angle of $5.\ 03^0$

$$N = (34.\ 64).\ (45830)/ \{(6.\ 28).\ (1.\ 94(10^7))\}$$
$$= 0.\ 013$$

For the transmitted frequency of 4440 Hz we get the N value to be 0. 016. These values are less than 'threshold' value of 2 suggesting that the process taking place in generation of 2-4 Hz side band spacing corresponds to linear amplification.

Test of non-linearity: Parameter of linearity

As is clear from Equation(8), and Inan(1977), Interaction length (L_{int}) depends upon a parameter, β which is function of loss cone pitch angle (α_0), wave frequency(f) and electron's gyro frequency(f_H)

$$\beta =\{ 3 +[\tan^2\alpha_0\ (f_H - f)/f_H)]/2 \tag{9}$$

Inan(1977) and Inan et el(1978) have provided a parameter (ρ) based on wave and geomagnetic field in homogeneity values which can be adopted to test whether linear processes are taking place or not. This parameter depends upon k, tan α, Ω_{Bw}, and $\partial\omega_H / \partial z$ such that

$$\rho = [k.\ \tan\alpha.\ \Omega_{Bw}] / [\beta.\ \partial\omega_H / \partial z]$$

where

$$\partial\omega_H / \partial z = 9.\ \omega_H.\ L_{int} / (LR_E)^2 \quad (10)$$

LR_E is the geocentric distance of the interaction region. We find, from Table 2 that the value of this parameter, ρ<1, though for non-linear mechanism it must be >1. Thus the 2-4 Hz sideband generation is a linear amplification mechanism.

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List of symbols and Abbreviations

L McIlwain parameter,

n	Density of electrons
f_H	Electrons gyro frequency
f	Frequency
μ	Refractive index of the medium
VR	Resonant velocity of electron
Ω(Bw)	Angular frequency of wave (trapping) magnetic field
Lint	Trapping length
ωT	Trap oscillatio frequency
k	Wave number

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