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Isotropic Non-Heisenberg Magnet for Spin *S=1*

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Abstract

Equations describing isotropic non-Heisenberg model are obtain by the method of generalized coherent states in a real parameterization. We linearization equation of motion near the ground state and obtain the equation of spin wave near this point.

Introduction

Many condensed matter systems can be successfully described with the help of effective continuum field models. In systems with reduced spatial dimensionality, topologically nontrivial field configurations are known to play an important role [1]. Magnetic systems are usually modeled with the help of the Heisenberg exchange interaction [2,3].

However, for spin $S > 1/2$ the general isotropic exchange goes beyond the purely Heisenberg interaction bilinear in spin operators $\vec{S}_i$, and may include higher-order terms of the type $\left(\vec{S}_i \vec{S}_j\right)^n$ with n up to *2S*. Particularly, a general *S = 1* model with the isotropic nearest-neighbor exchange on a lattice is described by the Hamiltonian

$$H = -\sum J\left(\vec{S}_i \vec{S}_{i+1}\right) + K\left(\vec{S}_i \vec{S}_{i+1}\right)^2 \qquad (1)$$

Where S_i^x, S_i^y, S_i^z are the spin operators acting at a site i, and $J, K > 0$ are respectively the bilinear (Heisenberg) and biquadratic exchange constant. The model (1) has been discussed recently in connection with *S = 1* bosonic gases in optical lattices [4] and in the context of the deconfined quantum criticality [5, 6].

The article is organized as following. In first we obtain coherent states in real parameterization for two group SU(2) and SU(3). In order to obtain semiclassical equation of motion we must obtain average values of spin, in this order, in section three we obtain these averages. In section four classical lattices Hamiltonian is obtain. In section five, with use of Hamiltonian, we calculate classical equations of motions. In final we linearization equation of motion near the ground state and obtain the equation of spin wave near this point.

Coherent states in SU(2) and SU(3) groups

Coherent state in SU(2) group is[7]

$$\begin{aligned}
|\psi\rangle &= \exp(-i\kappa S^z)\exp(-i\theta S^y)|0\rangle \\
&= C_0|0\rangle + C_1|1\rangle, \\
C_0 &= \cos(\theta/2)\exp(-i\varphi/2), \\
C_1 &= \sin(\theta/2)\exp(i\varphi/2)
\end{aligned} \tag{2}$$

And coherent state in SU(3) group is [8]

$$|\psi\rangle \quad = C_0|0\rangle + C_1|1\rangle + C_2|2\rangle \tag{3}$$

Where

$$\begin{aligned}
&C_0 = \exp(i\varphi)\times \\
&\left(\exp(-i\gamma)\sin^2(\theta/2)\cos g + \exp(i\gamma)\cos^2(\theta/2)\sin g\right) \\
&C_1 = (\sin\theta/2^{1/2})\left(\exp(-i\gamma)\cos g + \exp(i\gamma)\sin g\right) \\
&C_2 = \exp(-i\varphi)\times \\
&\left(\exp(-i\gamma)\cos^2(\theta/2)\cos g + \exp(i\gamma)\sin^2(\theta/2)\sin g\right)
\end{aligned} \tag{4}$$

Two angle, θ and φ, define the orientation of the classical spin vector. The angle γ is the rotation of the quadrupole moment about the spin vector. The parameter, g, defines change of the spin vector magnitude and that of the quadrupole moment.

Averaged spin operators and their products in SU(3) group

Here we consider classical counterparts of the spin operators and their products contained in the Hamiltonian (1). The vector

$$\vec{S} = \langle\psi|\hat{\vec{S}}|\psi\rangle \tag{5}$$

Can be regarded as a classical spin vector, and

$$Q^{ij} = \langle \psi | \hat{S}^i \hat{S}^j | \psi \rangle \tag{6}$$

as a component of the quadrupole moment. Because the spin operators at different lattice sites commute, we have for all such products

$$\langle \psi | \hat{S}^i_n \hat{S}^j_{n+1} | \psi \rangle = \langle \psi | \hat{S}^i_n | \psi \rangle \langle \psi | \hat{S}^j_{n+1} | \psi \rangle \tag{7}$$

where $|\psi\rangle = |\psi\rangle_n |\psi\rangle_{n+1}$.

Average spin expression for the SU(2) group is

$$\begin{aligned} \langle S^+ \rangle &= \exp(i\varphi)\sin\theta \\ \langle S^- \rangle &= \exp(-i\varphi)\sin\theta \\ \langle S^z \rangle &= \cos\theta \end{aligned} \tag{8}$$

and corresponding expressions for the SU(3) group are in the following form:

$$\begin{aligned} \langle S^+ \rangle &= \exp(i\varphi)\cos 2g \sin\theta \\ \langle S^- \rangle &= \exp(-i\varphi)\cos 2g \sin\theta \\ \langle S^z \rangle &= \cos 2g \cos\theta \end{aligned} \tag{9}$$

Classical lattice Hamiltonian

In this part, we derive classical lattice Hamiltonians which are obtain from Hamiltonian (1), averaged over coherent states (2) and (3). As was already mentioned the spin operators at neighboring sites commute, so the coherent state of the whole lattice is

$$|\psi\rangle = \prod_n |\psi\rangle_n \tag{10}$$

Averaging equation (1) with relation (10) and using equations (5-9), we obtain classical continuous limit of Hamiltonian in SU(2) group in following form

$$\begin{aligned} H = -\int \frac{dx}{a}\{J + K - \\ -(a^2/2)(J+2K)(\theta_x^2 + \varphi_x^2 \sin^2\theta)\} \end{aligned} \tag{11}$$

And classical Hamiltonian in SU(3) group is

$$\begin{aligned}H = &-\int \frac{dx}{a}\{J\cos^2 2g + K\cos^4 2g\\ &- a(g_x J\sin 4g + 4g_x K\cos^3 2g\sin 2g)\\ &- (a^2/2)(4g_x^2\cos^2 2g(J+2K\cos 4g)+\\ &\cos^2 2g(J+2K\cos^2 2g)(\theta_x^2+\varphi_x^2\sin^2\theta))\}\end{aligned} \tag{12}$$

In above relation if we set $g = 0$, we obtain classical Hamiltonian in SU(2) group.

Classical equations

In other to obtain classical equations of motion we set the above classical Hamiltonian in classical equations that obtained from Lagrangian in that group.

5-1) classical equation in SU(2) group

$$\begin{aligned}\varphi_t &= a_0^2(J+2K)(\varphi_x^2\cos\theta + \theta_{xx}\csc\theta)\\ \theta_t &= a_0^2(J+2K)\varphi_{xx}\sin\theta\end{aligned} \tag{13}$$

5-2) classical equation in SU(3) group

$$\begin{aligned}&\varphi_t = a_0^2(J\cos 2g + 2K\cos^3 2g)\times\\ &\times(\varphi_x^2\cos\theta + \theta_{xx}\csc\theta)\\ &\theta_t = a_0^2(J\cos 2g + 2K\cos^3 2g)\varphi_{xx}\sin\theta\\ &g_t = 0\\ &\gamma_t = -2J\cos 2g - 4K\cos^3 2g\\ &- a\{J(g_{xx}\cos 2g + 2g_x\cos 4g\csc 2g)\\ &+ K(2g_{xx}\cos^3 2g + 4g_x\cos^4 2g\csc 2g\\ &-12g_x\cos^2 2g\sin 2g)\} + (a_0^2/2)\{J(4g_x^2\cos 2g\\ &+\varphi_x^2\cos 2g - 2g_{xx}\cos 2g\cot 2g +\\ &\theta_{xx}\cos 2g\cot\theta + K(24g_x^2\cos^3 2g\\ &+2\varphi_x^2\cos^3 2g\sin^2\theta - 4g_{xx}\cos^3 2g\cot 2g\\ &+4g_{xx}\cos^3 2g\sin 2g - 8g_x^2\cos 2g\sin^2 2g\\ &+2\theta_{xx}\cos^3 2g\cot\theta)\}\end{aligned} \tag{14}$$

Classical ground state and minimum of energy

Note that to use the equation (14) in investigation of ferromagnets with isotropic non-Heisenberg term, it is necessary to find the classical ground states of this magnet. To this end, we consider only a term in Hamiltonian (12) that without a derivative:

$$H_0 = -\int \frac{dx}{a}(J\cos^2 2g + K\cos^4 2g) \tag{15}$$

In the ferromagnetic J and K>0, to find the smallest value of the H_0 we vary it respect to all the parameter, the ground state is obtain in the points

$$g = 0 \quad or \quad g = \pi/4 \tag{16}$$

And minimum of energy is

$$H_0 = -(1/a)(J + K) \tag{17}$$

Linearize equations of motion near ground state for SU(3) group

We consider now the dispersion of spin waves propagating near the ground states. To do this end, we linearize the classical equations in (14) near the ground states.

Classical Hamiltonian (12) near the ground state is

$$\begin{aligned} &H = -\int \frac{dx}{a}\{J + K - \\ &-(a^2/2)((J + 2K)(\theta_x^2 + \varphi_x^2 \sin^2\theta))\} \end{aligned} \tag{18}$$

And equations are

$$\begin{aligned} &\varphi_t = a_0^2(J + 2K)(\varphi_x^2 \cos\theta + \theta_{xx} \csc\theta) \\ &\theta_t = a_0^2(J + 2K)\varphi_{xx} \sin\theta \\ &g_t = 0 \\ &\gamma_t = -a_0^2(J + 2K)(\varphi_x^2 \cos^2\theta + \theta_{xx} \cot\theta) \end{aligned} \tag{19}$$

For the ground state, near the point $\theta = \pi/2$, in linear small excitation the above equation changes in the following form:

$$\begin{aligned} &\varphi_t = a_0^2(J + 2K)\theta_{xx} \to (1/\omega_0)\varphi_t = m\theta_{xx} \\ &\theta_t = a_0^2(J + 2K)\varphi_{xx} \to (1/\omega_0)\theta_t = m\varphi_{xx} \\ &g_t = 0 \\ &\gamma_t = 0 \end{aligned} \tag{20}$$

We consider now the dispersion of spin wave propagating near the ground state. To this end we considering two functions θ and φ in the following plane waves,

$$\begin{aligned} &\varphi = \varphi_0 \exp(i(\omega t - kx)) + \overline{\varphi}_0 \exp(-i(\omega t - kx)) \\ &\theta = \theta_0 \exp(i(\omega t - kx)) + \overline{\theta}_0 \exp(-i(\omega t - kx)) \end{aligned} \tag{21}$$

We obtain the following equation for the spin wave propagating near the ground state

$$\omega^2 = m^2 k^4 \omega_0^2 \tag{22}$$

Where the value of m for SU(3) group is $m = (J + 2K)a_0^2$. It is evidence from equation (22) that the quadrupole branch for the Hamiltonian (1) is nondispersive.

Discussion

In terms of spin coherent states we have investigated S = 1 spin quantum system with the bilinear and biquadratic isotropic exchange in the continuum limit. The proper Hamiltonian of the model can be written as bilinear on the generators of SU(3) group[9] . Knowledge of such group structure enables us to obtain some new exact analytical results. The analysis of the proper classical model allows us to get different soliton solutions with finite energy and the spatial distribution of spin-dipole and/or spin-quadrupole moments termed as dipole, quadrupole, and dipole-quadrupole soliton, respectively.

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