International Journal of Physics and Applications.
ISSN 0974-3103 Volume 3, Number 2 (2011), pp. 205-208

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Even Graceful Labeling of the Union of Paths and Cycles

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Abstract

M. Ibrahim Moussa proved that the graph $C_m \cup P_n$ is odd graceful if m is even, he also described an algorithm to label the vertices and the edges of the vertex set $V(C_m \cup P_n)$ and the edge set $E(C_m \cup P_n)$. In this paper we studied the graph $C_m \cup P_n$ is even graceful if m is odd with some certain condition.

Keywords: Graceful labeling, graceful graph, even graceful labeling.

Introduction

If the vertices are assigned values subject to certain conditions then it is known as graceful labeling. The study of graceful graphs and graceful labeling methods was introduced by Rosa [1].

A function f is called graceful labeling of graph G if f: $V \to \{0, 1, 2, \ldots..q\}$ is injective and the induced function $f^* : E \to \{1, 2, \ldots\ldots q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective. A graph which admits graceful labeling is called graceful graph.

A graph G of size q is odd graceful or even graceful if there is an injection f from V(G) to $\{0,1,2,3,4,\ldots\ldots\ldots\ldots 2q - 1\}$ such that when each edge uv is assigned the label $|f(u) - f(v)|$, the resulting edge labels are $\{1,3,5,\ldots..2q - 1\}$ or $\{2,4,6, \ldots\ldots 2q - 2\}$ respectively.

Even Graceful of $C_m \cup P_n$

Theorem

If K is a given integer and m = 2k+1, then the graph $C_m \cup P_n$ is even graceful for every n = m+2.

Proof

Let $V(C_m) = \{u_1, u_2, \ldots\ldots u_m\}$, $V(P_n) = \{ v_1, v_2, \ldots\ldots v_n\}$ where $V(C_m)$ is the vertex set of the cycle Cm and $V(P_n)$ is the vertex set of path P_n and $q = (n+m)$.

For every u_i and v_i we defined the even graceful labeling functions $f(u_i)$ and $f(v_i)$ respectively as follows:

Vertex labeling of C_m

$f(u_1) = 0$ $\quad f(u_2) = 2q - 2$ $\quad f(u_3) = 2$ $\quad f(u_4) = 2q - 4$

$f(u_5) = 4$ $\quad f(u_6) = 2q - 6 \ldots\ldots\ldots$ $\quad f(u_{i-1})$ = zero and even $\quad f(u_i) = 2q - i$

Vertex labeling of P_n

$$f(vi) = \begin{cases} i & i = 0,2,4,\ldots\ldots \\ (q+2) - i & i = 0,2,4,\ldots\ldots \end{cases}$$

The first value of $f(v_i)$ denoted the labeling of the vertex which are given below and the second value of $f(v_i)$ denoted the labeling of above vertices of the path P_n.

The number of vertices in the path P_n arranged as follows: The number of above vertices is (k+2) and the number of below vertices is (k+1).

If k is even each $(C_m \cup P_n)$ graph we just neglect the labeling (6+n), n = 2, 4, 6, ……. in the path P_n. If k is odd we eliminate (6+n), n= multiples of 6 from each $(C_m \cup P_n)$ graph.

Edge labeling of C_m

$f^*(u_1u_2) = 2q - 2$ $\quad f^*(u_2u_3) = 2q - 4$ $\quad f^*(u_3u_4) = 2q - 6$

$f^*(u_{m-1}u_m) = 2q - 2p$ $\quad f^*(u_mu_1) = 2q - 4m + p$, where p=0,2,4, ….

Edge labeling of P_n

$f^*(v_iv_{i+1}) = 2q - 2m - p$, ………..2 except the labeling $(2q - 4m + p)$ since that labeling belongs to the C_m.

These conditions shows that each component in the given graph has even graceful, the path is even graceful, the cycle with an odd number of vertices is even graceful. We have to prove that the vertex labels are distinct and all the edge labels are even numbers {2, 4, 6, ………..2q - 2}.

In this theorem, the conditions which are mentioned above for odd and even value of k, to prove the distinction of vertex and edge labeling. Otherwise it cannot form the graceful graph.

This theorem is illustrated by the following examples.

Example

Let $V(C_5) = \{u_1, u_2, u_3, u_4, u_5\}$, $V(P_n) = V(P_{m+2}) = V(P_7) = \{ v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$

Where $V(C_5)$ is the vertex set of the cycle C_5 and $V(P_7)$ is the vertex set of the path P_7 and q = n+m = 12. Number of vertices above the path is 3 and below the path 4.the even graceful labeling functions $f(u_i)$ and $f(v_i)$ as given below.

$f(u_1) = 0$ $f(u_2) = 22$ $f(u_3) = 2$ $f(u_4) = 20$ $f(u_5) = 4$

Here we neglect the vertex labeling 6 from the path P_7.

$f(v_1) = 0$ $f(v_2) = 14$ $f(v_3) = 2$ $f(v_4) = 12$ $f(v_5) = 4$

$f(v_6) = 10$ $f(v_7) = 8$

The edge labeling function f* defined as follows,

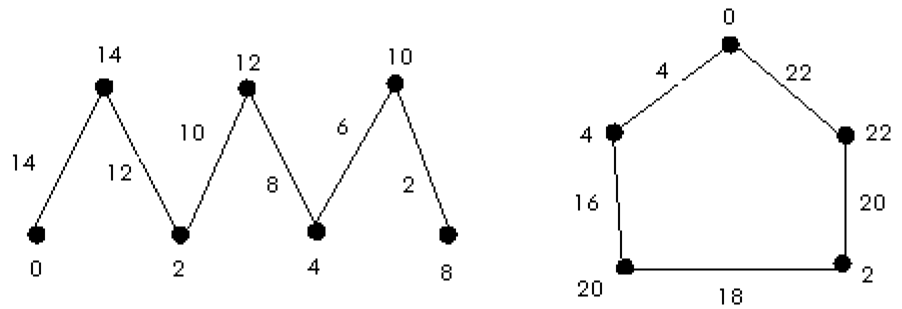
$f^*(u_1u_2) = 22$ $f^*(u_2u_3) = 20$ $f^*(u_3u_4) = 18$

$f^*(u_4u_5) = 16$ $f^*(u_5u_1) = 4$.

and

$f^*(v_1v_2) = 14$ $f^*(v_2v_3) = 12$ $f^*(v_3v_4) = 10$

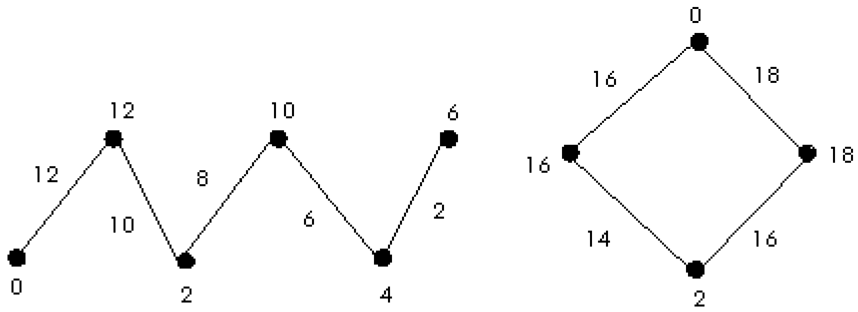
$f^*(v_4v_5) = 8$ $f^*(v_5v_6) = 6$ $f^*(v_6v_7) = 2$



Clearly we conclude that all the vertex labeling and edge labeling are even graceful labeling. Therefore this graph is even graceful graph.

Example

Consider the graph $(C_4 \cup P_6)$ where m = 4 and q = 10. Using the above theorem the vertex labeling of the cycle is $f(u_1) = 0$ $f(u_2) = 18$ $f(u_3) = 2$ $f(u_4) = 16$. The corresponding edge labeling is $f^*(u_1u_2) = 18$ $f^*(u_2u_3) = 16$ $f^*(u_3u_4) = 14$ $f^*(u_4u_1) = 16$



Clearly the above cycle is not graceful and the above and below arrangement of the path P_n is not suitable for this path P_6.

Therefore the theorem is only true for the odd value of m.

Conclusion

In this paper, we explicitly proved a theorem of even graceful labeling of the graph $(Cm \cup Pn)$ for m is odd. Also we found this theorem is not true for m is even.

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