International Journal of Physics and Applications.
ISSN 0974-3103 Volume 3, Number 2 (2011), pp. 209-218

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Higher Order Amplitude Squeezing in Seventh Harmonic Generation

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Abstract

The quantum effect of squeezing of electromagnetic field is investigated in fundamental mode in seventh harmonic generation under the short time approximation. Squeezing is investigated in higher orders of field amplitude in fundamental mode. Squeezing is found to be dependent on coupling constant g and phase of the field amplitude. The effect of photon number on squeezing and signal to noise ratio in higher order of field amplitude in fundamental mode has also been investigated.

Keywords: Harmonic generation; Squeezing; Nonlinear optics.

Introduction

Optical fields in states with purely quantum mechanical properties are the key ingredient of quantum optics. Light fields presenting squeezing are well known examples of non-classical states. As a direct consequence of the quantum nature of electromagnetic radiations, it is well known that all light fields fluctuate. The minimum uncertainty states having less fluctuation in one quadrature than coherent state at the expense of increased fluctuations in the other quadrature are called squeezed states of the electromagnetic field.

Initial studies regarding squeezing were done keeping in view the theoretical aspects [1-3]. The first experimental demonstration of squeezed light succeeded in

1985 [4]. These states have nonclassical noise statistics and their predicted generation schemes include as harmonic generation [5], multi-wave mixing processes [6], optical parametric oscillation [7-8], and nonlinear polarisation rotation [9].

Recent work has highlighted the potential applications of squeezed states such as in the high precision quantum measurement and the processing of quantum information. They have been used to improve the sensitivity of interferometers for the detection of gravitational waves [10-11]. Another field of application is quantum teleportation [12-13] and quantum computation [14]. Recently squeezed states of light have been used for quantum information networks [15-16].

Definition of squeezing and higher order squeezing

Squeezed states of an electromagnetic field are the states with reduced noise below the vacuum limit in one of the canonical conjugate quadrature. Amplitude-squared squeezing is defined in terms of operators Y_1 and Y_2 as

$$Y_1 = \frac{1}{2}\left(A^2 + A^{\dagger 2}\right) \quad \text{and} \quad Y_2 = \frac{1}{2i}\left(A^2 - A^{\dagger 2}\right)$$

where Y_1 and Y_2 are the real and imaginary parts of the square of field amplitude, respectively. A and $A^\dagger$ are slowly varying operators defined by

$$A = ae^{i\omega t} \quad \text{and} \quad A^\dagger = a^\dagger e^{-i\omega t}.$$

The operators Y_1 and Y_2 obey the commutation relation

$$\left[Y_1, Y_2\right] = i\,(2N + 1)$$

which leads to the uncertainty relation

$$\Delta Y_1 \Delta Y_2 \geq \left\langle \left(N + \frac{1}{2}\right) \right\rangle$$

where N is the usual number operator.

Amplitude-squared squeezing is said to exist in Y_i variable if

$$(\Delta Y_i)^2 < \left\langle \left(N + \frac{1}{2}\right) \right\rangle \quad \text{for } i = 1 \ or \ 2$$

Amplitude-cubed squeezing is defined in terms of operators

$$Z_1 = \frac{1}{2}\left(A^3 + A^{\dagger 3}\right) \quad \text{and} \quad Z_2 = \frac{1}{2i}\left(A^3 - A^{\dagger 3}\right)$$

The operators Z_1 and Z_2 obey the commutation relation

$$\left[Z_1, Z_2\right] = \frac{i}{2}(9N^2 + 9N + 6)$$

which leads to the uncertainty relation

$$\Delta Z_1 \Delta Z_2 \geq \frac{1}{4}(9N^2 + 9N + 6)$$

Amplitude-cubed squeezing exists when

$$(\Delta Z_i)^2 < \frac{1}{4}\left\langle(9N^2 + 9N + 6)\right\rangle \quad \text{for } i = 1 \ or \ 2$$

Squeezing of fundamental mode in seventh harmonic generation

Seventh harmonic generation model is shown in Figure 1. In this model, the interaction is looked upon as a process which involves the absorption of seven photons, each having a frequency ω_1 going from state |1> to state |2> and emission of one photon of frequency ω_2, where $\omega_2 = 7\omega_1$.

The Hamiltonian for this process is given as follows ($\hbar$=1)

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + g\left(a^7 b^\dagger + a^{\dagger 7} b\right) \tag{1}$$

in which g is a coupling constant for seventh harmonic generation.

A = a $\exp(i\omega_1 t)$ and B = b $\exp(i\omega_2 t)$ are the slowly varying operators at frequencies ω_1 and ω_2, a($a^\dagger$) and b($b^\dagger$) are the usual annihilation (creation) operators, respectively. The Heisenberg equation of motion for fundamental mode A is given as ($\hbar$=1)

$$\frac{\partial A}{\partial t} = \frac{dA}{dt} + i\left[H, A\right] \tag{2}$$

Using Eq. (1) in Eq. (2), we obtain

$$\dot{A} = -7igA^{\dagger 6}B \tag{3}$$

Similarly,

$$\dot{B} = -igA^7 \tag{4}$$

By assuming the short time ($\approx 10^{-12}$ s) interaction of waves with the medium and expanding A(t) by using Taylors series expansion and retaining the terms up to g^2t^2 as

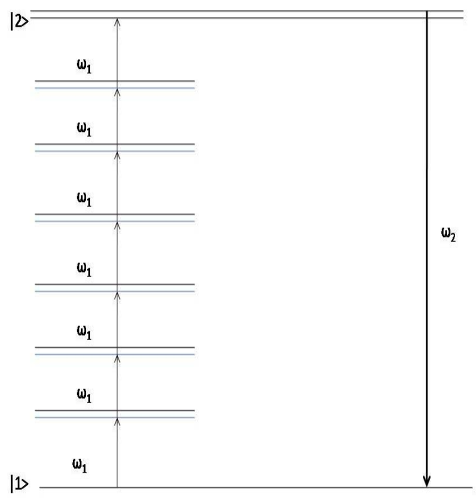

Figure 1: Seventh harmonic generation model.

$$A\left(t\right) = A - 7igtA^{\dagger 6}B + \frac{7}{2}g^2t^2\Big[42\Big(A^{\dagger 5}A^6 + 15^{\dagger 4}A^5 + 100A^{\dagger 3}A^4 + 300A^{\dagger 2}A^3 + 360A^{\dagger}A^2 + 120A\Big)B^{\dagger}B - A^{\dagger 6}A^7\Big] \tag{5}$$

Initially, we consider the quantum state of the field amplitude as a product of coherent state for the fundamental mode *A* and the vacuum state for the harmonic mode *B* i.e.

$$\left|\psi\right\rangle = \left|a\right\rangle\left|0\right\rangle \tag{6}$$

Using Eq.(5) and (6), the second order amplitude in fundamental mode is expressed as

$$A^2\left(t\right) = A^2 - 7igt\left(2A^{\dagger 6}A + 6A^{\dagger 5}\right)B - \frac{7}{2}g^2t^2\left(2A^{\dagger 6}A^8 + 6A^{\dagger 5}A^7\right) \tag{7}$$

and

$$A^{\dagger 2}\left(t\right) = A^{\dagger 2} + 7igt\left(2A^{\dagger}A^6 + 6A^5\right)B^{\dagger} - \frac{7}{2}g^2t^2\left(2A^{\dagger 8}A^6 + 6A^{\dagger 7}A^5\right) \tag{8}$$

For amplitude squared squeezing, the real quadrature component for the fundamental mode is given as

$$Y_{1A}(t) = \frac{1}{2}\left[A^2(t) + A^{\dagger 2}(t)\right] \tag{9}$$

Using Eqs.(6), (7) and (9), we get the expectation values as

$$\begin{aligned}\left\langle\psi\left|Y_{1A}^2(t)\right|\psi\right\rangle = \frac{1}{4}\Big[&\alpha^4 + \alpha^{*4} + 2|\alpha|^4 + 4|\alpha|^2 + 2 - 7g^2t^2(2\alpha^4|\alpha|^{12} \\ &+ 18\alpha^4|\alpha|^{10} + 60\alpha^4|\alpha|^8 + 60\alpha^4|\alpha|^6 + 2\alpha^{*4}|\alpha|^{12} \\ &+ 18\alpha^{*4}|\alpha|^{10} + 60\alpha^{*4}|\alpha|^8 + 60\alpha^{*4}|\alpha|^6 + 4|\alpha|^{16} + 16|\alpha|^{14})\Big]\end{aligned} \tag{10}$$

and

$$\begin{aligned}\left\langle\psi\left|Y_{1A}(t)\right|\psi\right\rangle^2 = \frac{1}{4}\Big[&\alpha^4 + \alpha^{*4} + 2|\alpha|^4 - 7g^2t^2(2\alpha^4|\alpha|^{12} + 6\alpha^4|\alpha|^{10} \\ &+ 2\alpha^{*4}|\alpha|^{12} + 6\alpha^{*4}|\alpha|^{10} + 4|\alpha|^{16} + 12|\alpha|^{14})\Big]\end{aligned} \tag{11}$$

Therefore,

$$\begin{aligned}\left[\Delta Y_{1A}(t)\right]^2 = \frac{1}{4}\Big[&4|\alpha|^2 + 2 - 7g^2t^2(12\alpha^4|\alpha|^{10} + 60\alpha^4|\alpha|^8 + 60\alpha^4|\alpha|^6 \\ &+ 12\alpha^{*4}|\alpha|^{10} + 60\alpha^{*4}|\alpha|^8 + 60\alpha^{*4}|\alpha|^6 + 4|\alpha|^{14})\Big]\end{aligned} \tag{12}$$

Using Eqs.(5) and (6), number of photons in mode A may be expressed as

$$\begin{aligned}N_{1A}(t) &= A^\dagger(t)A(t) \\ &= A^\dagger A - 7igt\left(A^{\dagger 6}B - A^6B^\dagger\right) - 7g^2t^2A^{\dagger 7}A^7\end{aligned} \tag{13}$$

$$\begin{aligned}N_{1A}^2(t) &= N_{1A}(t)N_{1A}(t) \\ &= A^{\dagger 2}A^2 + A^\dagger A - 7g^2t^2\left(2A^{\dagger 8}A^8 + 7A^{\dagger 7}A^7\right)\end{aligned} \tag{14}$$

Using Eqs.(6) and (13), we get

$$\left\langle N_{1A}(t) + \frac{1}{2}\right\rangle = \frac{1}{4}\left[4|\alpha|^2 + 2 - 28g^2t^2|\alpha|^{14}\right] \tag{15}$$

Subtracting Eq.(15) from Eq.(12), we get

$$\left[\Delta Y_{1A}(t)\right]^2 - \left\langle N_{1A}(t) + \frac{1}{2}\right\rangle = -15.5g^2t^2\left(|\alpha|^{14} + 5|\alpha|^{12} + 5|\alpha|^{10}\right)\cos 4\theta \tag{16}$$

The right hand side of Eq.(16) is negative and thus shows the existence of squeezing in the second order of the fundamental mode for all values of θ for which $\cos 4\theta > 0$.

Using Eqs.(5) and (6) the third-order amplitude of the fundamental mode is expressed as

$$A^3(t) = A^3 - 7igt\left(3A^{\dagger 6}A^2 + 18A^{\dagger 5}A + 30A^{\dagger 4}\right)B - \frac{7}{2}g^2t^2\left(3A^{\dagger 6}A^9 + 18A^{\dagger 5}A^8 + 30A^{\dagger 4}A^7\right) \tag{17}$$

and

$$A^{\dagger 3}(t) = A^{\dagger 3} + 7igt\left(3A^{\dagger 2}A^6 + 18A^{\dagger}A^5 + 30A^4\right)B^{\dagger} - \frac{7}{2}g^2t^2(3A^{\dagger 9}A^6 + 18A^{\dagger 8}A^5 + 30A^{\dagger 7}A^4\Big) \tag{18}$$

Using Eq.(17) the real quadrature component for third-order squeezing in fundamental mode is expressed as

$$\begin{aligned} Z_{1A}(t) &= \frac{1}{2}\left[A^3(t) + A^{\dagger 3}(t)\right] \\ &= \frac{1}{2}\Big[A^3 + A^{\dagger 3} - 7igt\left(3A^{\dagger 6}A^2 + 18A^{\dagger 5}A + 30A^{\dagger 4}\right)B + 7igt\Big(3A^{\dagger 2}A^6 + 18A^{\dagger}A^5 \\ &+ 30A^4\Big)B^{\dagger} - \frac{7}{2}g^2t^2\Big(3A^{\dagger 6}A^9 + 18A^{\dagger 5}A^8 + 30A^{\dagger 4}A^7 + 3A^{\dagger 9}A^6 + 18A^{\dagger 8}A^5 \\ &+ 30A^{\dagger 7}A^4)\Big] \end{aligned} \tag{19}$$

Using Eqs.(6) and (19), we get the expectation values as

$$\begin{aligned} \left\langle\psi\left|Z_{1A}^2(t)\right|\psi\right\rangle &= \frac{1}{4}\Big[\alpha^6 + \alpha^{*6} + 2|\alpha|^6 + 9|\alpha|^4 + 18|\alpha|^2 + 6 - 7g^2t^2\Big(3\alpha^6|\alpha|^{12} \\ &+ 45\alpha^6|\alpha|^{10} + 300\alpha^6|\alpha|^8 + 900\alpha^6|\alpha|^6 + 1080\alpha^6|\alpha|^4 + 360\alpha^6|\alpha|^2 \\ &+ 3\alpha^{*6}|\alpha|^{12} + 45\alpha^{*6}|\alpha|^{10} + 300\alpha^{*6}|\alpha|^8 + 900\alpha^{*6}|\alpha|^6 + 1080\alpha^{*6}|\alpha|^4 \\ &+ 360\alpha^{*6}|\alpha|^2 + 6|\alpha|^{18} + 54|\alpha|^{16} + 132|\alpha|^{14}\Big)\Big] \end{aligned} \tag{20}$$

and

$$\begin{aligned} \left\langle\psi\left|Z_{1A}(t)\right|\psi\right\rangle^2 &= \frac{1}{4}\Big[\alpha^6 + \alpha^{*6} + 2|\alpha|^6 - 7g^2t^2(3\alpha^6|\alpha|^{12} + 18\alpha^6|\alpha|^{10} \\ &+ 30\alpha^6|\alpha|^8 + 3\alpha^{*6}|\alpha|^{12} + 18\alpha^{*6}|\alpha|^{10} + 30\alpha^{*6}|\alpha|^8 \\ &+ 6|\alpha|^{18} + 36|\alpha|^{16} + 60|\alpha|^{14})\Big] \end{aligned} \tag{21}$$

Therefore,

$$\begin{aligned} \left[\Delta Z_{1A}(t)\right]^2 &= \frac{1}{4}\Big[9|\alpha|^4 + 18|\alpha|^2 + 6 - 7g^2t^2\{(27|\alpha|^{10} + 270|\alpha|^8 + 900|\alpha|^6 \\ &+ 1080|\alpha|^4 + 360|\alpha|^2 + 60)(\alpha^6 + \alpha^{*6}) + 18|\alpha|^{16} + 72|\alpha|^{14}\}\Big] \end{aligned} \tag{22}$$

Using Eqs.(13) and (14), we get

$$\frac{1}{4}\left\langle 9N_{1A}^{2}\left(t\right)+9N_{1A}\left(t\right)+6\right\rangle =\frac{1}{4}[9\left|\alpha\right|^{4}+18\left|\alpha\right|^{2}+6-7g^{2}t^{2}(18\left|\alpha\right|^{16}+72\left|\alpha\right|^{14})] \tag{23}$$

Subtracting Eq. (23) from Eq. (22), we obtain

$$\left[\Delta Z_{1A}\left(t\right)\right]^{2}-\frac{1}{4}\left\langle 9N_{1A}^{2}\left(t\right)+9N_{1A}\left(t\right)+6\right\rangle =-7g^{2}t^{2}(13.5\left|\alpha\right|^{16}+135\left|\alpha\right|^{14}+450\left|\alpha\right|^{12}+540\left|\alpha\right|^{10}+180\left|\alpha\right|^{8})\cos 6\theta \tag{24}$$

The right hand side of Eq.(24) is negative, indicating that squeezing occurs in cubed amplitude for all values of θ for which cos6θ >0 in the fundamental mode of seventh harmonic generation.

Signal-to-noise ratio

Signal to noise ratio is defined as ratio of the magnitude of the signal to the magnitude of the noise. With the approximations θ=0 and $|gt|^2 \ll 1$, the maximum signal to noise ratio (in decibels) in field amplitude and higher orders, is given below.

Using Eqs.(11) and (12), SNR in amplitude-squared squeezing is given as

$$SNR_{2}=20^{*}\log_{10}\frac{\left(2\left|\alpha\right|^{6}+6\left|\alpha\right|^{4}\right)}{\left(7\left|\alpha\right|^{4}+30\left|\alpha\right|^{2}+30\right)} \tag{25}$$

Using Eqs.(21) and (22), SNR in amplitude-cubed squeezing is expressed as.

$$SNR_{3}=20^{*}\log_{10}\frac{\left(\left|\alpha\right|^{10}+6\left|\alpha\right|^{8}+10\left|\alpha\right|^{6}\right)}{\left(6\left|\alpha\right|^{8}+51\left|\alpha\right|^{6}+150\left|\alpha\right|^{4}+180\left|\alpha\right|^{2}+60\right)} \tag{26}$$

Results

The results show the presence of squeezing in second and third order of field amplitude in fundamental mode in seventh harmonic generation. We denote right hand side of Eqs.(16) and (24) by S_Y and S_Z respectively, which shows the presence of squeezing in amplitude-squared and amplitude-cubed states of the field.Taking $|gt|^2 = 10^{-4}$ and θ=0 for maximum squeezing, the variations of S_Y and S_Z are shown in Figures 2 and 3 respectively. Degree of squeezing is shown as a function of $|\alpha|^2$.

It is clear from Figures 2-3, that the squeezing increases non-linearly. with $|\alpha|^2$. This confirms that the squeezed states are associated with the photon number in fundamental mode. The variation of SNR in different orders of field amplitude for a squeezed state with photon number has also been shown in Figure 4. The signal-to-noise ratio is higher in lower order squeezed states as reported earlier for Raman Process [17].

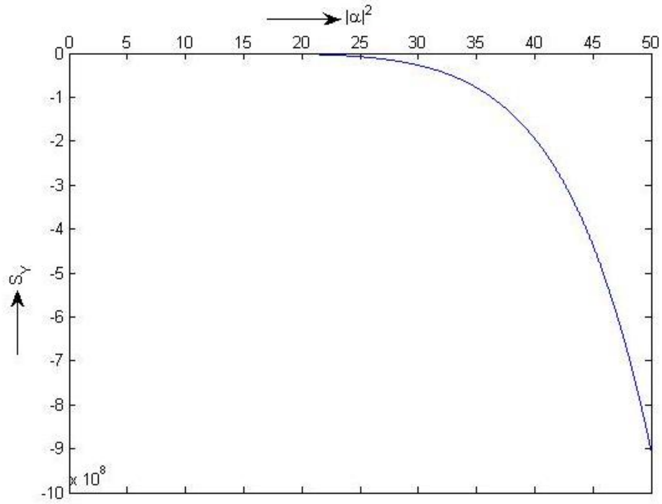

Figure 2: Dependence of amplitude-squared squeezing on $|\alpha|^2$.

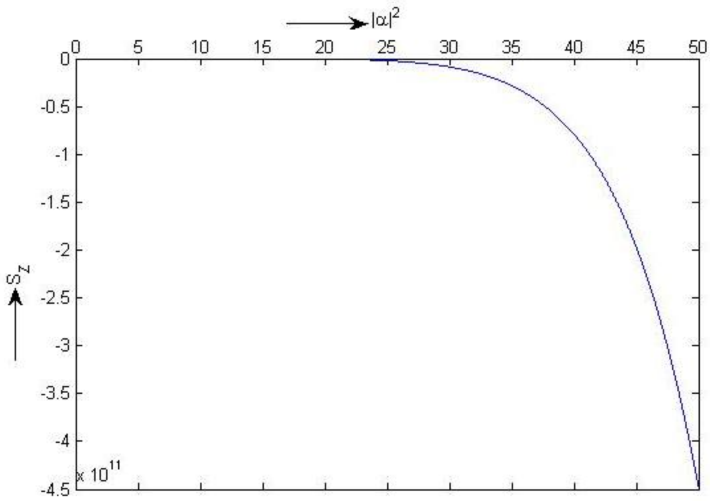

Figure 3: Dependence of amplitude-cubed squeezing on $|\alpha|^2$.

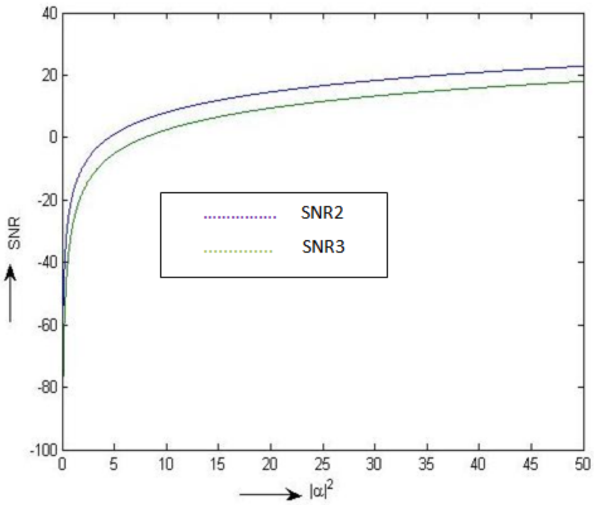

Figure 4: Signal to noise ratio for second and third order squeezing.

Conclusion

It is shown that the selective phase values of field amplitude of fundamental mode during seventh harmonic generation lead to squeezing in second and third order of field amplitude which can be used in the field of high-precision measurements, quantum information and in reducing noise in the output of certain non-linear devices. Further, Figures 2 and 3, show that the degree of squeezing increases with increase in the order of field amplitude of the fundamental mode.

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