International Journal of Physics and Applications.
ISSN 0974-3103 Volume 3, Number 3 (2011), pp. 259-263

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Role of Shell Corrections in Magic Nuclei

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Abstract

The reason for the development of semi empirical mass formula is that if it is accurate enough, it can be used to predict masses and therefore possible decay modes and decay energies of unexplored atoms. Seeger introduced a shell correction term, which is a function of proton number and neutron number. This term enables the explanation of magic nuclei by predicting various parameters e.g. separation energies of protons, separation energies of neutrons etc.

Keywords: Semi empirical mass formula, Magic nuclei, shell effects, separation energy

Introduction

The landscape of nuclear binding energy spanned in the co-ordinate system of proton and neutron is well structured as evident by the measured binding energies. At the magic proton or neutron numbers 2, 8, 20, 28, 50 and 82 the nuclei have an increased binding energy relative to average trend. For neutron, N=126 is also identified as a 'magic number'. Among other special properties, the doubly magic nuclei are spherical and resist deformation.

The stability of the heaviest and super heavy elements has been a long-standing fundamental question in nuclear science. Theoretically, the mere existence of the heavy elements with Z >104 is entirely due to the quantal shell effects.

If the heaviest nuclei were governed by the classical liquid drop model, they would fission immediately from their ground states due to the large electric charge. However in the mid 1960s, with the invention of the shell-correction method, it was realized that atomic numbers could exist due to the strong shell stabilization [1], [2], [3]. Most of the heaviest elements found recently are believed to be well deformed. Many nucleus structure models have been proposed, including, Liquid drop model, Collective model, Shell model. None of these could completely explain experimental data of nuclear structure. The nuclear radius (R) is considered to be one of basic things that any model must explain for stable nuclei.

Liquid Drop Model and Shell Correction

Semi empirical mass formula given by Von Weizsacker [4] can be used to predict accurately the masses of nuclei which ranges from light nuclei to heavy nuclei .In reality this situation is complicated. The inability of the liquid drop model proposed by Bohr and Wheeler [5] to account for the observed asymmetry in the mass yield curve of binary fission was demonstrated by Cohen and Swiatecki [6]. It does not explain the peaks in Binding energy curve at certain key values of N and Z.

There might be local variation of masses due to effects known as shell effects. Introduction of shell correction explains magicity in the binding energy curve. A.E.L.Deperink[7] has shown that if in addition to an improved version of liquid drop mass formula with modified symmetry and coulomb terms, shell effects are modelled, a very simple formula is obtained with a rms deviation from the 2003 database of atomic masses of about 800keV.

Methodology

Two nucleon separation energies are difference of binding energies. They provide information on the relative stability of the nuclei. G.G.Bunatyan [8] has studied the behaviour of change of slope of two-nucleon separation energies and has shown that the maximum slope, at closed shells is due to Wigner energy.

It is shown by V.Yu. Denisov[9] that position of deep local minima of shell correction associated with magic numbers in the region of super heavy nuclei depend on the parameters of central spin-orbital mean-field potentials.

The separation energies of two protons for Odd Z and Even N nuclei have been calculated using the formula

$$S_{2p} = B(Z,N) - B(Z-2,N) \qquad 1$$

The Binding energies have been calculated by using semi empirical formula.

$$E_B = a_v A - a_s A^{\frac{2}{3}} - \frac{a_c Z(Z-1)}{A^{\frac{1}{3}}} - \frac{a_A (A-2Z)^2}{A} + \delta(A,Z) \qquad 2$$

The values of these coefficients are are calculated by “Wapstra”[10] as

$$a_v = 14.1MeV \; a_s = 13MeV a_c = .595MeV \; a_A = 19MeV \qquad 3$$

Due to pairing, a nucleus with an even number of protons is tightly bound than odd number of proton nucleus [11]. As we are taking Odd nucleons, therefore pairing term is taken as zero.

The Binding energies of elements having atomic numbers from 1 to 112 with all their possible isotopes have been calculated. Then we calculate the separation energy of two protons as well as one proton for Odd Z and Even N nuclei. Fig. 1 is a plot of separation energies of two protons as a function of Z (S_{2P}).It is observed that this curve is not in agreement with the experimental plot.

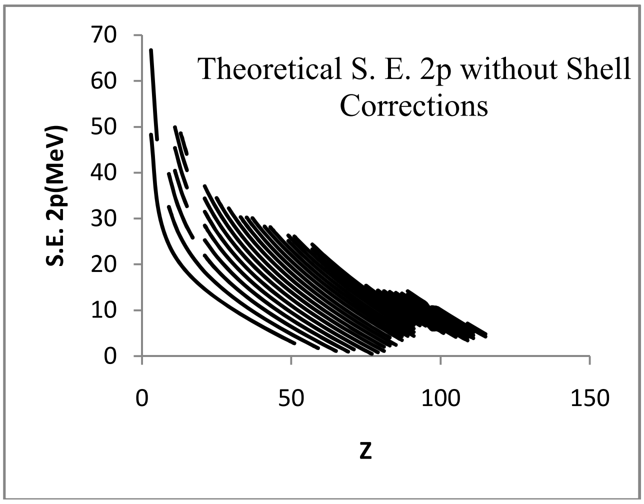

Figure 1: Plot of theoretical values of separation energies of two protons with Z without shell corrections

This problem is resolved by taking in account the shell effects. Seeger gave a formula for calculating binding energies [12]

$$\Delta M_0(Z,A) = 7.2887Z + 8.0713(A-Z) - \alpha A + 0.8076Z^2A^{-\frac{1}{3}}\left(1 - 0.7636Z^{-\frac{2}{3}} - 2.29A^{-\frac{2}{3}}\right) + \gamma A^{\frac{2}{3}} + \left(\beta - \eta A^{-\frac{1}{3}}\right)A^{-1}[(A-2Z)^2 + 2|A-2Z|] - S_{jk}(N',Z') \qquad 4$$

The last term in the above equation is a shell correction term, which is the function of parameter N and Z defined as

$$N' = \frac{N-N_j}{N_{j+1}-N_j}$$
$$Z' = \frac{Z-Z_k}{Z_{k+1}-Z_k} \qquad 5$$

Here N_j and Z_k are magic numbers

N_j, Z_k=8, 20, 50, 82, 126, 184 and $N_j \leq N < N_{j+1}$ $Z_k \leq Z < Z_{k+1}$

Thus the function $S_{jk}$is different for different intervals between magic numbers. The formula for S_{jk} is

$$S_{jk}(N',Z') = \xi_j sinN'\pi + \xi_k sinZ'\pi + \nu_j sin2N'\pi + \nu_k sin2Z'\pi$$
$$+\left(\phi_{j+}\phi_k\right)(sinN'\pi)(sinZ'\pi) + \chi \qquad 6$$

The adjustable constants have been determined by method of least squares. The constants are the same for the full range of masses listed from A =19 to A =260.

α= 17.06 MeV β = 33.61 MeV γ= 25.00 MeV η =59.54 MeV

In our calculation we use Von Weisacker's formula [4] to calculate the binding energy as

$$\Delta M(Z,A) = a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{\frac{1}{3}}} - \frac{a_A (A-2Z)^2}{A} - S_{jk}(N',Z') \quad 7$$

Using this formula, separation energies for one proton are calculated for all the possible isotopes from Z= 70 to Z=112. Now these separation energies include shell corrections. The experimental values of separation energies of one proton are taken from the table compiled by Audi and Wapstra [10] and separation energies of one proton are calculated and plotted against Z. From comparison of Fig 2 and Fig 3, we see the change of slope at Z=81 proving the magic character of Z=82.

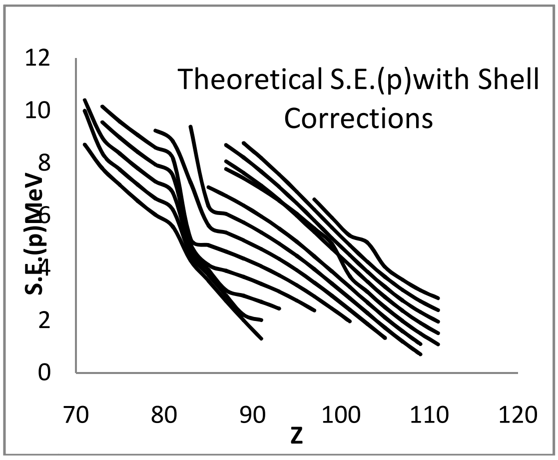

Figure 2: Plot of theoretical values of separation energies of proton with Z with shell corrections

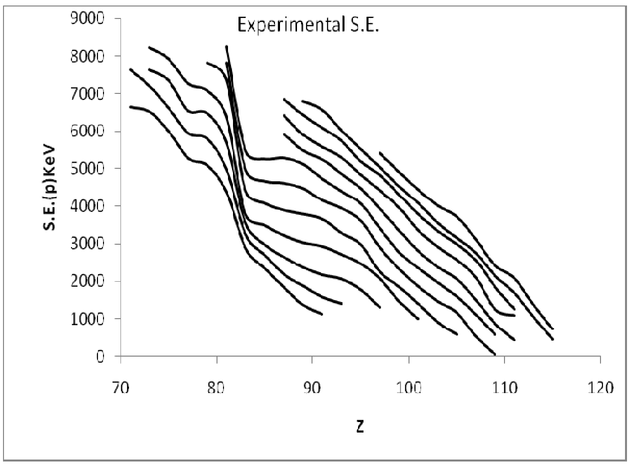

Figure 3: Plot of experimental values of separation energies of proton with Z

Conclusion

Liquid drop model of the nucleus structure explains all the binding energies but cannot explain the existence of magic nuclei. Shell corrections play very important role in nuclear structure. By following this methodology we can guess the next magic nucleus in super heavy elements.

Acknowledgement

The authors thank the D A V College, Amritsar for the support rendered in doing this theoretical work.

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