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Choosing Fundamental Constants

A. Tan and M.E. Edwards

Alabama A & M University
P.O. Box 447, Normal, AL 35762, U. S. A.
E-mail: arjun.tan@aamu.edu

Abstract

Candidates for the fundamental constants of nature include the universal gravitational constant G, the speed of light in vacuum c, the electronic charge e and the reduced Planck's constant $\hbar$. Planck pointed out that since the fine structure constant relates c, e and $\hbar$, only two of them can be fundamental and at least one of them has to be derived. Sommerfeld considered e to be derived whereas Dirac argued that $\hbar$ was more likely to be derived. It is postulated in this study that in order for three constants to be fundamental, they must yield the fundamental units of mass M, length L and time T in terms of them. Three plausible options are arrived at: (1) G, c & e; (2) G, e & $\hbar$; and (3) G, c & $\hbar$. Since e and $\hbar$ are related by the Bohr radius, Option 2 is ruled out. Further, since c and $\hbar$ are both related to the photon, Option 3 is also ruled out. By the method of elimination, the fundamental constants of nature are deemed to be those prescribed by Option 1, viz., G, c and e. This conclusion is not in conflict with Dirac's assessment.

Keywords: Fundamental constants; Fundamental units; Dimensional Analysis.

Introduction

There are a handful of physical quantities which can be considered as fundamental constants of nature. They include the universal gravitational constant G, the speed of light c, the electronic charge e, and the reduced Planck's constant $\hbar$. Prescribed by Newton's law of gravitation, G pervades the large-scale universe; e is responsible for electromagnetic interactions and is important in the intermediate range scale; $\hbar$ is related to quantum effects and is all-important in the nano-scale; whereas c is constant for all occasions. The constancy of c is a postulate of the special theory of relativity and $\hbar$ appears in Heisenberg's and Schroedinger's formulations of quantum mechanics. Combinations of these constants also appear in a variety of phenomena in atomic physics, e.g., in the fine-structure constant, Rydberg's constant, the Bohr magneton and the nuclear magneton.

Before the advent of quantum mechanics, Johnstone-Stoney was the first to identify G, c and e as the fundamental constants of nature [1]. After the discovery of the fine-structure constant, expressed in Gaussian units as $\alpha = e^2/\hbar c \approx 1/137$, it was recognized that three of the constants c, e & $\hbar$ were related and thus could not be all fundamental at the same time [2]. Planck [2] first suggested that either e or $\hbar$ must be derived. Sommerfeld [3] considered that e was derived, whereas Dirac [4] argued that $\hbar$ was more likely to be derived. Recently, Tomilin [5] suggested that since the fine-structure constant in SI units $\alpha = e^2/4\pi\varepsilon_0\hbar c$ (ε_0 = permittivity of free space) connects four constants (c, e, $\hbar$ & ε_0), c, e & $\hbar$ can, after all, be all fundamental. It is safe to state that the question as to which are the real fundamental constants, remains unsettled.

In this paper, we offer an alternative approach to choosing the fundamental constants. We propose to select three fundamental constants out of the four candidates of G, c, e & $\hbar$. We stipulate that for the three constants to be fundamental, the fundamental units of mass (M), length (L) and time (T) must be uniquely expressible in terms of the former. There are four plausible options: (1) c, e & $\hbar$; (2) G, c & e; (3) G, e & $\hbar$; and (4) G, c & $\hbar$.

Method

We first express the dimensions of G, c, e & $\hbar$ in terms of M, L and T, which is entirely possible in the Gaussian system of units [6]:

$$\boldsymbol{G} = \frac{\boldsymbol{L}^3}{\boldsymbol{M}\boldsymbol{T}^2} \tag{1}$$

$$\boldsymbol{c} = \frac{\boldsymbol{L}}{\boldsymbol{T}} \tag{2}$$

$$\boldsymbol{e} = \frac{\sqrt{\boldsymbol{M}\boldsymbol{L}^3}}{\boldsymbol{T}} \tag{3}$$

and

$$\hbar = \frac{\boldsymbol{M}\boldsymbol{L}^2}{\boldsymbol{T}} \tag{4}$$

Next, we proceed with the inverse process of expressing M, L & T in terms of the constants G, c, e & $\hbar$ for all the above options. This can be done methodically by dimensional analysis [7]. We illustrate the procedure with two examples.

Example 1. Option 1. Express M in terms of c, e & $\hbar$. Let $\boldsymbol{M} = \boldsymbol{c}^{\boldsymbol{i}}\hbar^{\boldsymbol{j}}\boldsymbol{e}^{\boldsymbol{k}}$.
From Eqs. (2) - (4), we get

$$\boldsymbol{M} = \boldsymbol{M}^{\boldsymbol{j}+\boldsymbol{k}/2}\boldsymbol{L}^{\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k}/2}\boldsymbol{T}^{-\boldsymbol{i}-\boldsymbol{j}-\boldsymbol{k}} \tag{5}$$

We must have

$$\boldsymbol{j} + \frac{\boldsymbol{k}}{2} = 1 \tag{6}$$

$$\boldsymbol{i} + 2\boldsymbol{j} + \frac{3\boldsymbol{k}}{2} = 0 \tag{7}$$

and
$$\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k} = 0 \tag{8}$$

Eqs. (6) – (8) are a set of linear equations in *i*, *j* & *k*, which has solutions only if the determinant Δ of the coefficients of *i*, *j* & *k*, is non-zero. But here

$$\Delta = \begin{vmatrix} 1 & 1 & 1/2 \\ 1 & 1 & 3/2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Hence the solution does not exist. Thus, *M* cannot be expresses in terms of *c*, *e* & *ħ*. The same holds true for *L* and *T*.

Example 2. Option 2. Express *M* in terms of *G*, *c* & *e*. Let $\boldsymbol{M} = \boldsymbol{c}^{i}\boldsymbol{G}^{j}\boldsymbol{e}^{k}$.
From Eqs. (1) – (3), we get

$$\boldsymbol{M} = \boldsymbol{L}^{\boldsymbol{i}+3\boldsymbol{j}+3\boldsymbol{k}/2}\boldsymbol{M}^{-\boldsymbol{j}+\boldsymbol{k}/2}\boldsymbol{T}^{-\boldsymbol{i}-2\boldsymbol{j}-\boldsymbol{k}} \tag{9}$$

We must have

$$\boldsymbol{i} + 3\boldsymbol{j} + \frac{3\boldsymbol{k}}{2} = 0 \tag{10}$$

$$-\boldsymbol{j} + \frac{\boldsymbol{k}}{2} = 1 \tag{11}$$

and
$$\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k} = 0 \tag{12}$$

Eqs. (10) – (12), when solved simultaneously, give $i = 0$; $j = -1/2$; and $k = 1$. Thus, $\boldsymbol{M} = \boldsymbol{e}/\sqrt{\boldsymbol{G}}$. Likewise, *L* and *T* can be expressed in terms of *G*, *c* & *e* in Option 2, as well as the relevant quantities in Options 3 and 4. The results are entered in Table 1.

Table 1. Fundamental Units in terms of Fundamental Constants

Option	Constants	*M*	*L*	*T*
Option 1	*c*, *e* & *ħ*	-	-	-
Option 2	*G*, *c* & *e*	$\frac{\boldsymbol{e}}{\sqrt{\boldsymbol{G}}}$	$\frac{\sqrt{\boldsymbol{G}}\boldsymbol{e}}{\boldsymbol{c}^2}$	$\frac{\sqrt{\boldsymbol{G}}\boldsymbol{e}}{\boldsymbol{c}^3}$
Option 3	*G*, *e* & *ħ*	$\frac{\boldsymbol{e}}{\sqrt{\boldsymbol{G}}}$	$\frac{\sqrt{\boldsymbol{G}}\hbar^2}{\boldsymbol{e}^3}$	$\frac{\sqrt{\boldsymbol{G}}\hbar^3}{\boldsymbol{e}^5}$
Option 4	*G*, *c* & *ħ*	$\sqrt{\frac{\boldsymbol{c}\hbar}{\boldsymbol{G}}}$	$\sqrt{\frac{\boldsymbol{G}\hbar}{\boldsymbol{c}^3}}$	$\sqrt{\frac{\boldsymbol{G}\hbar}{\boldsymbol{c}^5}}$

Discussion

Since the fundamental units cannot be derived from the fundamental constants in Option 1, this option must be ruled out as unfeasible. In other words, c, e & $\hbar$ cannot be chosen as the fundamental constants of nature. This is in agreement with the assertion of Planck [2] and thus one must choose G and two from c, e & $\hbar$. At this point Options 2, 3 & 4 are still viable.

In order to decide which option is the best choice, we note that the Bohr radius $\boldsymbol{a}_0 = \hbar^2 / \boldsymbol{m}_e \boldsymbol{e}^2$ (which can be interpreted as the characteristic radius of the hydrogen atom in its ground state) relates the two constants e and $\hbar$, plus the electron mass m_e. As m_e and a_0 are both likely to be constants, one of e and $\hbar$ is possibly less than fundamental, which is in agreement with the assertions of Sommerfeld [3] and Dirac [4]. Hence Option 3 is also ruled out.

Of the two remaining options, we note that c and $\hbar$ are both related to the photon (its energy and propagation) and perhaps they can, somehow, be connected. Using customary notations, we have from the wave relation ($c = \nu\lambda$) and Planck's law ($E = h\nu$): $hc = E\lambda$. Even though the relation between $\hbar$ and c is weaker than that between e and $\hbar$, one of $\hbar$ and c is possibly less than fundamental. Thus Option 4 is also deemed to be less than desirable. Note that the fundamental units M, L & T in this option are the Planck mass, Planck length and Planck time, respectively, and Option 4 may be considered Planck's own choice of fundamental constants.

Finally, Option 2 is the only option left standing. There appears to be no possible dependence between any two of G, c & e, and they are our logical choice for the fundamental constants of nature. This is identical to the original choice of Johnstone-Stoney [1] and is also not in conflict with Dirac's assessment [4].

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