International Journal of Physics and Applications.
ISSN 0974-3103 Volume 5, Number 1 (2013), pp. 7-12

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Spectra and Decay Properties of S-Wave Bottomonium States

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Abstract

In this paper we calculated the masses of S-wave mesons, pseudoscalar and vector decay constants of bottomonium based on the relativistic harmonic confinement model for quarks and antiquarks, The quark potential model used for investigation contains the scalar plus vector harmonic potential interaction, the residual two-body coulomb interaction and the spin dependent interaction. The spin-spin interactions are perturbatively incorporated with the confinement energy to get the respective meson masses. Here, we have also calculated the decay properties of ground state bottomonium. The results are being compared with values obtained from other theoretical models and experimental values

Keywords: semi-relativistic harmonic model, confined one gluon exchange potential, decay constants, S-wave spectrum.

Introduction

The spectroscopy and decay rates of quarkonia are quite important to study due to huge amount of high precession data acquired from many experimental facilities world over[1-3], which have been continuously providing accurate information about hadrons particularly in heavy quark flavour sector[4, 5].On the theoretical side, the study of heavy quarkonia provides information related to QCD at hadronic scale and checks the ability of perturbative QCD calculations [6]. Since the exact confinement form of QCD is not known, one has to go for phenomenological models are either non-relativistic quark model(NRQCD) or relativistic quark model(RQM).The investigation of hadronic properties composed of heavy quarks and anti quarks gives a very important insight into the better understanding of the constituent masses. In this context, we reconsider the ϒ(nS) states to study their properties. The spectroscopic parameters deduced using the phenomenological approach will be employed to compute the decay properties such as the leptonic and hadronic decaywidths for the ground state with no additional parameters. for this we have use the confinement model developed in the spirit of RHM [7-13].

In this paper, we present the computed masses of S-wave mesons as well as the decay constant of heavy flavour combinations, in the frame work of semi-relativistic quark model(SRQM) by including the coulomb potential as short range non-perturbative gluon effect in addition to the conventional COGEP derived from QCD.

Methodology

The Model For the present study of heavy-heavy bound state systems such as $b\bar{b}$.We consider a confinement scheme for Hamiltonian as

$$H = V_{CONF}(\vec{r}_{ij}) + V_{COUL}(\vec{r}_{ij}) + V_{SD}(\vec{r}_{ij}) \tag{1}$$

According to confinement scheme based on RHM, the confinement potential of the quark/antiquark in a mesonic system is given by [10].

$$V_{CONF}(\vec{r}) = \frac{1}{2}(1+\gamma_0)A^2 r^2 + M \tag{2}$$

where A^2 is the quark strength with γ_0 is the Dirac matrix with and M is the quark mass. They have a different value for each quark. The eigen value of eqn.(1) is given by [7, 10,].

$$E_n^2 = M^2 + (2n+1)\Omega_n^2 \tag{3}$$

The total energy of the hadron is obtained by adding the individual contribution of quarks.

The residual coulomb potential is calculated perturbatively in the confinement basis as

$$V_{COUL}(\vec{r}) = e_s^{eff}(\mu)\frac{1}{r} \tag{4}$$

Here $e_s^{eff}(\mu)$ is the effecting strong coupling constant defined as in [13].The spin dependent part of the usual one gluon exchange potential[13] contains three types of interaction term, such as spin-spin, the spin-orbit and the tensor part given by[14].

$$V_{SD}(\vec{r}) = V_{SS}(\vec{r}) + V_{LS}(\vec{r}) + V_T(\vec{r}) \tag{5}$$

The spin-orbit term $V_{LS}(\vec{r})$ and the tensor term $V_T(\vec{r})$ describes the fine structure of mesonic states. while spin-spin term $V_{SS}(\vec{r})$ gives spin singlet-triplet hyperfine splitting. The coefficient of these spin-dependent terms due OGEP can be written as.

$$V_{COGEP}^{SS}(\vec{r_{ij}}) = \frac{\alpha_s}{4} N^4 \lambda_i.\lambda_j [D_0(r) + \frac{1}{(E+M)^2}[4\pi\delta^3(r) - c^4 r^4 D_1(r)][1 - \frac{2}{3}\sigma_i.\sigma_j]] \tag{6}$$

$$V_{COGEP}^{LS}(\vec{r_{ij}}) = \frac{\alpha_s}{4} \frac{N^4}{(E+M)^2} \lambda_i.\lambda_j \frac{1}{2r}[\{(r \times (p_i - p_j).(\sigma_i.\sigma_j))(D_0'(r) + 2D_1'(r))\}$$
$$+ \{(r \times (p_i + p_j).(\sigma_i - \sigma_j))(D_0'(r) - D_1'(r))\}] \tag{7}$$

$$V_{COGEP}^{T}(\vec{r_{ij}}) = -\frac{\alpha_s}{4} \frac{N^4}{(E+M)^2} \lambda_i.\lambda_j \left[\frac{D_1''}{3} - \frac{D_1'}{3r}\right] \hat{S}_{ij} \tag{8}$$

Where $\hat{S}_{ij} = [3(\sigma_i.\hat{r})(\sigma_j.\hat{r}) - (\sigma_i - \sigma_j)]$. Both spin-orbit and tensor forces affect states with L>0.The $Q\bar{Q}$ wave function for each meson is expressed in terms of oscillator wave functions corresponding to CM and relative coordinates. The oscillator quantum number for the CM wavefunctions are restricted to $N_{CM} = 0$. the Hilbert space of relative wavefunctions is truncated at radial quantum number $n = 5$.The Hamiltonian matrix is constructed in the basis states of $[N_{CM} = 0,\ L_{CM} = 0; n^{2S+1}L_j >$ and diagonolised. the diagonal values give the masses of the ground state and radically excited states

Decay Properties of Bottomonium States

Apart from the masses of the low lying states, the hyperfine splits due to chromomagnitic interaction and the right behaviuor of the wavefunction that provides as the correct predictions of the decay rates are important features of any successful model.

Accordingly, the wave functions of the identified nS states of quarkonia ($b\bar{b}$) are obtained. The decay constants for the pseudoscalar and vector mesons are given by the Van Royan and Weisskopf formula [15, 16].

$$f_{p/v}^2 = 12 \frac{|\psi_{p/v}(0)|^2}{m_{p/v}} \tag{9}$$

Where, $\Psi_{p/v}(0)$ is the wave function at the origin of the pseudoscalar or vector mesons and $m_{p/v}$ is the ground state mass of the pseudoscalar or vector mesons.

The leptonic decay width of $\Upsilon(nS) \to l^+ l^-$ are computed as [17]

$$\Gamma^{l^+l^-} = \frac{16\pi\alpha_e^2 e_Q^2}{m_V^2}\left|\psi^2(0)\right|^2\left(1-\frac{16}{3\pi}\alpha_S\right) \tag{10}$$

The decay widths with radiative correction for $\Upsilon \to ggg$ and $\Upsilon \to \gamma gg$ are given respectively by [17]

$$\Gamma^{ggg} = \frac{40(\pi^2-9)\alpha_s^3}{81 m_q^2}\left|\psi^2(0)\right|^2\left(1-\frac{4.9}{\pi}\alpha_S\right) \tag{11}$$

$$\Gamma^{\gamma gg} = \frac{32(\pi^2-9)\alpha_e\alpha_s^2 e_q^2}{9 m_q^2}\left|\psi^2(0)\right|^2\left(1-\frac{7.4}{\pi}\alpha_S\right) \tag{12}$$

α_e is the electromagnetic coupling constant and α_s is the strong coupling constant. The parameters of the model are the mass of the b-quark(m_b) and the oscillator size parameter.$b_n(=1/\Omega_n)$, the strong coupling constant α_s.the values of parameters used in our calculation are m_b= 4.67GeV, α_s= 0.3, $\alpha_e = 1/137$ and the value of b is found to be 0.34 For $b\bar{b}$ mesons $e_Q = -1/3$ .The predicted results of bottomonia are tabulated in Table I and II respectively.

Table 1. Results of $b\bar{b}$ mass in comparison with PDG and other models

nS	Mass (MeV) [our]	Mass (MeV) Expt.[4]	Mass (MeV) [18]	*nS*	Mass (MeV) [our]	Mass (MeV) Expt.[4]	Mass (MeV) [18]
Υ(1S)	9461.52	9460	9460.38	η_b(1S)	9394.43	9391	9392.91
Υ(2S)	10022.59	10023	10023.3	η_b(2S)	9989.18	-	9987.42
Υ(3S)	10347.81	10355	10364.2	η_b(3S)	10326.51	-	10333.9
Υ(4S)	10573.86	10579	10636.4	η_b(4S)	10558.85	-	10609.4
Υ(5S)	10753.83	10860	-	η_b(5S)	10741.79	-	-

Table 2. Decay Properties

observable	our	Expt.[4]	others
$f_p(\eta_b(1S))$	355 MeV	-	599 MeV[15]
$f_v(\Upsilon(1S))$	356 MeV	708 ± 8 MeV	665 MeV[15]
$\Gamma_{e^+e^-}(\Upsilon(1S))$	0.323 keV	1.340 ± 0.018 keV	1.35 keV[19]
$\Gamma_{ggg}(\Upsilon(1S))$	10.3 keV	-	-
$\Gamma_{\gamma gg}(\Upsilon(1S))$	0.24 keV	-	-

Results and Discussion

Based on the relativistic harmonic confinement scheme we have been able to predict the bottonomium S- wave mass states, which are in good agreement with the reported PDG values as compared to the predicted values of [18] we have also predicted $\eta_b(1S-5S)$ states within the mass range 9.9GeV-10.741GeV. As expected, it is seen from the from table 1 that the masses increases as *nS* goes from 1S to 5S, such a behavior is seen even in reported experimental values except Υ(10860).Thus we identify Υ(10860) as either a mixed state or exotic state. With no additional parameters, we have been able to predict the lepton decay widths of Υ(nS) states. which are in good agreement with the known experimental values[4]. And we have also predicted the lepton decay widths of $\Gamma_{ggg}(\Upsilon(1S))$ = 10.3 keV and $\Gamma_{\gamma gg}(\Upsilon(1S))$ = 0.24 keV. We hope to find future experimental confirmation in favor of our predictions.

In conclusion, a simple relativistic harmonic confinement scheme with the residual coulomb potential employed in the present study is found to be quite successful in predicting various properties of $b\bar{b}$ mesons. The method can be useful to study various hadronic and radiative transitions of charm-beauty system.

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