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Theoretical Design of Picoseconds Fabry–Pérot Filter and Study the Dispersion using Coupled Mode Equation

Elham Jasim Mohammad

Al-Mustansiriyah University, Collage of Sciences,
Physics Department.
E-mail: elham.jasim@yahoo.com

Abstract

Fabry-Pérot interferometers or etalons are used in optical modems, spectroscopy, lasers, and astronomy.In this paper we used the coupled mode equation to design the Fabry–Pérot filter and study the picosecond dispersion, where, the picosecond is 10^{-12} of a second. That is one trillionth, or one millionth of one millionth of a second, or 0.000 000 000 001 seconds.Coupled mode analysis is widely used in the field of integrated optoelectronics for the description of two coupled waves traveling in the same direction. The program is written in MATLAB to simulate and analysis the Fabry–Pérot properties.

Index Terms: Fabry–Pérot filter, Coupled ModeTheory, Coupling Coefficient, Finesse.

1. Introduction

In optics, a Fabry–Pérot interferometer or etalon is typically made of a transparent plate with two reflecting surfaces, or two parallel highly reflecting mirrors. (Technically the former is an etalon and the latter is an interferometer, but the terminology is often used inconsistently.) Its transmission spectrum as a function of wavelength exhibits peaks of large transmission corresponding to resonances of the etalon. It is named after Charles Fabry and Alfred Perot. "Etalon" is from the French *étalon*, meaning "measuring gauge" or "standard".

The resonance effect of the Fabry–Pérot interferometer is identical to that used in a dichroic filter. That is, dichroic filters are very thin sequential arrays of Fabry–Pérot interferometers, and are therefore characterised and designed using the same mathematics.

Etalons are widely used in telecommunications, lasers and spectroscopy to control and measure the wavelengths of light. Recent advances in fabrication technique allow the creation of very precise tunable Fabry–Pérot interferometers [1,2].

2. Coupled Mode Theory and Computing the Coupling Coefficient and Dispersion

Coupled mode analysis is widely used in the field of integrated optoelectronics for the description of two coupled waves traveling in the same direction (co-directional coupling) or in the opposite direction (contra directional coupling). Indeed, it is the method of choice for weakly index-modulated systems like waveguides in general. For such systems the coupled-mode approach represents an excellent approximation to the exact problem. Additionally, a lot of powerful analytical design tools based on the coupled mode equations have been developed. They allow the design of various types of structures and the fast calculation of their spectral response characteristics. On the other hand, for the design of optical filters and mirrors, which are composed of discrete layers with large differences in the refractive indices (e.g., dielectric multilayer coatings), the coupled-mode approach is hardly considered. Its applicability seems to be questionable because the assumption of a small perturbation is violated in the case of large index discontinuities.

In fact, for such systems the index difference is of the same order of magnitude as the average and effective refractive index, respectively [3].

In coupled mode equations, $\kappa = \frac{\pi \Delta n}{2\lambda}$ defines the coupling coefficient for the first order refractive-index variation Δn and λ is the design wavelength. Using coupled wave equations, the boundary conditions can be written as:

$$E_f(0) = r_1 E_b(0) \qquad \textit{and} \qquad E_b(L) = r_2 E_f(L).$$

where r_1 and r_2 are the reflectivity's for the electric field at $z = 0$ and $z = L$, respectively [4].

When the evanescent wave in the dielectric layer is reflected by a non-absorbing metal, the group delay time is negative when the electric field vector is in the plane of incidence and positive when the electric field vector is perpendicular to the plane of incidence. Similarly, a frustrated Fabry-Perot interferometer shows negative group delay times for angles of incidence greater than specific p-wave and s-wave critical angles [5].

The group delay (GD) is defined as the negative of the derivative of the phase response with respect to frequency [6]. In physics and in particular in optics, the study of waves and digital signal processing, the term delay meaning: the rate of change of the total phase shift with respect to angular frequency [7,8]: $GD = -\frac{d\phi}{d\omega}$.Through a device or transmission medium, where ϕ is the total phase shift in radians, and ω is the angular frequency in radians per unit time, equal to $2\pi f$, where f is the frequency (hertz if delay is measured in seconds).The group delay dispersion (GDD) can be determined by the derivative of the delay with respect to the angular frequency ω and is given by [7,8]: $GDD = \frac{dGD}{d\omega}$.

3. Mathematical Model for the Fabry-Perot Filter

The heart of the Fabry–Pérot interferometer is a pair of partially reflective glass optical flats spaced millimeters to centimeters apart, with the reflective surfaces facing each other. (Alternatively, a Fabry–Pérot*etalon* uses a single plate with two parallel reflecting surfaces.) The flats in an interferometer are often made in a wedge shape to prevent the rear surfaces from producing interference fringes; the rear surfaces often also have an anti-reflective coating [1].

In a typical system, illumination is provided by a diffuse source set at the focal plane of a collimating lens. A focusing lens after the pair of flats would produce an inverted image of the source if the flats were not present; all light emitted from a point on the source is focused to a single point in the system's image plane. In the accompanying illustration, only one ray emitted from point A on the source is traced. As the ray passes through the paired flats, it is multiply reflected to produce multiple transmitted rays which are collected by the focusing lens and brought to point A' on the screen.

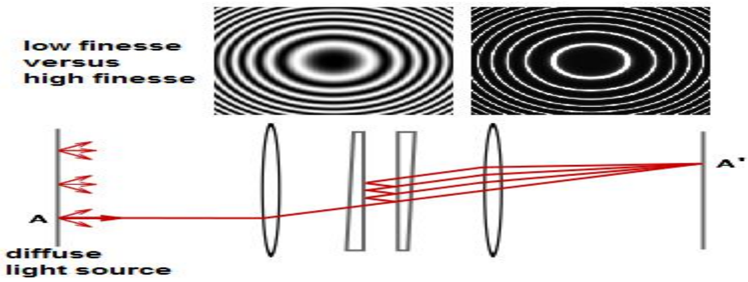
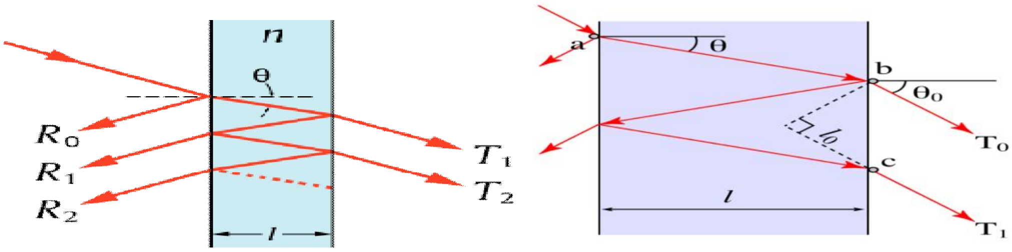


Figure 1: Fabry–Pérot interferometer, using a pair of partially reflective, slightly wedged optical flats. The wedge angle is highly exaggerated in this illustration; only a fraction of a degree is actually necessary. Low-finesse versus high-finesse images corresponds to mirror reflectivities of 4% (bare glass) and 95% [1].

The complete interference pattern takes the appearance of a set of concentric rings. The sharpness of the rings depends on the reflectivity of the flats. If the reflectivity is high, resulting in a high Q factor, monochromatic light produces a set of narrow bright rings against a dark background. A Fabry–Pérot interferometer with high Q is said to have high *finesse* [1].

The varying transmission function of an etalon is caused by interference between the multiple reflections of light between the two reflecting surfaces. Constructive interference occurs if the transmitted beams are in phase, and this corresponds to a high-transmission peak of the etalon. If the transmitted beams are out-of-phase, destructive interference occurs and this corresponds to a transmission minimum. Whether the multiply reflected beams are in phase or not depends on the wavelength λ of the light (in vacuum), the angle the light travels through the etalon θ, the thickness of the etalon ℓ and the refractive index of the material between the reflecting surfaces n.



Figures 2: A Fabry–Pérot etalon. Light enters the etalon and undergoes multiple internal reflections [1,2].

The phase difference between each succeeding reflection is given by δ [1]:

$$\delta = \left(\frac{2\pi}{\lambda}\right) 2n\ell\cos\theta \qquad \ldots(1)$$

If both surfaces have a reflectance R, the transmittance function of the etalon is given by [1]:

$$T_e = \frac{(1-R)^2}{1+R^2-2R\cos\delta} = \frac{1}{1+F\sin^2(\delta/2)} \qquad \ldots(2)$$

where: $F = \dfrac{4R}{(1-R)^2}$, is the *coefficient of finesse.*

Maximum transmission $T_e = 1$ occurs when the optical path length difference $2nl\cos\theta$ between each transmitted beam is an integer multiple of the wavelength. In the absence of absorption, the reflectance of the etalon R_e is the complement of the transmittance, such that $T_e + R_e = 1$. The maximum reflectivity is given by [1]:

$$R_{\max} = 1 - \frac{1}{1+F} = \frac{4R}{(1+R)^2} \qquad \ldots(3)$$

and this occurs when the path-length difference is equal to half an odd multiple of the wavelength.The wavelength separation between adjacent transmission peaks is called the free spectral range (FSR) of the etalon, $\Delta\lambda$, and is given by [1]:

$$\Delta\lambda = \frac{\lambda_0^2}{2n\ell\cos\theta + \lambda_0} \approx \frac{\lambda_0^2}{2n\ell\cos\theta} \qquad \ldots(4)$$

where λ_0 is the central wavelength of the nearest transmission peak. The FSR is related to the full-width half-maximum, $\delta\lambda$, of any one transmission band by a quantity known as the *finesse*: $f = \frac{\Delta\lambda}{\delta\lambda} = \frac{\pi}{2\arcsin(1/\sqrt{F})}$.This is commonly approximated for $R\rangle 0.5$ by [1]: $f \approx \frac{\pi\sqrt{F}}{2} = \frac{\pi R^{1/2}}{1-R}$ Etalons with high finesse show sharper transmission peaks with lower minimum transmission coefficients. In the oblique incidence case, the finesse will depend on the polarization state of the beam, since the value of R, given by the Fresnel equations, is generally different for p and s polarizations.

A Fabry–Pérot interferometer differs from a Fabry–Pérot etalon in the fact that the distance ℓ between the plates can be tuned in order to change the wavelengths at which transmission peaks occur in the interferometer. Due to the angle dependence of the transmission, the peaks can also be shifted by rotating the etalon with respect to the beam [1,2].

Two beams are shown in the diagram above. One of which T_0 is transmitted through the etalon, and the other of which T_1 is reflected twice before being transmitted. At each reflection, the amplitude is reduced by $\sqrt{R}$ and the phase is shifted by π, while at each transmission through an interface the amplitude is reduced by $\sqrt{T}$. Assuming no absorption, conservation of energyrequires $T+R=1$. In the derivation below, n is the index of refraction inside the etalon, and n_0 is that outside the etalon. The incident amplitude at point a is taken to be one, and phasors are used to represent the amplitude of the radiation. The transmitted amplitude at point b will then be: $t_0 = Te^{ik\ell/\cos\theta}$, where $k = 2\pi n/\lambda$ is the wavenumber inside the etalon and λ is the vacuum wavelength. At point c the transmitted amplitude will be: $T\,\mathrm{Re}^{2\pi i+3ik\ell/\cos\theta}$ [1,2].

The total amplitude of both beams will be the sum of the amplitudes of the two beams measured along a line perpendicular to the direction of the beam. The amplitude at point b can therefore be added to an amplitude t_1 equal in magnitude to the amplitude at point c, but retarded in phase by an amount $k_0\ell_0$ where $k_0 = 2\pi n_0/\lambda$ is the wavenumber outside of the etalon. Thus: $t_1 = RTe^{2\pi i+3ik\ell/\cos\theta - ik_0\ell_0}$, where ℓ_0 is: $\ell_0 = 2\ell\tan\theta\sin\theta_0$.

Neglecting the 2π phase change due to the two reflections, the phase difference between the two beams is: $\delta = \frac{2k\ell}{\cos\theta} - k_0\ell_0$. The relationship between θ and θ_0 is given by Snell's law: $n\sin\theta = n_0\sin\theta_0$. So that the phase difference may be written: $\delta = 2k\ell\cos\theta$. To within a constant multiplicative phase factor, the amplitude of the mth transmitted beam can be written as: $t_m = TR^m e^{im\delta}$.The total transmitted amplitude is the sum of all individual beams' amplitudes: $t = \sum_{m=0}^{\infty} t_m = T\sum_{m=0}^{\infty} R^m e^{im\delta}$ .The series is a geometric series whose sum can be expressed analytically. The amplitude can be rewritten as: $t = \frac{T}{1-\mathrm{Re}^{i\delta}}$.

The intensity of the beam will be just t times its complex conjugate. Since the incident beam was assumed to have an intensity of one, this will also give the transmission function [1,2]:

$$T_e = tt^* = \frac{T^2}{1+R^2-2R\cos\delta} \qquad \ldots(5)$$

4. Simulation Result and Discussion

MATLAB is a great and easy tool to use to simulate optical electronics. All the results below are got after following these steps:

1. Calculate the transmittance function, finesse and contrast factor of Fabry-Perot filter.
2. Implementation of the Transfer Matrix method for solution of Coupled Mode equations.
3. Found the phase difference to calculate the amplitude and power transmission coefficient of Fabry-Perot filter.
4. Calculate the delay and dispersion of Fabry-Perot filter in picoseconds units.
5. Found the POLYFIT for the delay and dispersion results.

Figure 3is about the transmitted intensity versus the interference order. It shows the transmittance function for different values of F. Instead of δ, the corresponding interference order $\delta/2\pi$ is noted. The mean, median, mode and the standard deviation (STD) are tablets in table .1 for five different data. Fig.4is about the finesse and the mirror reflectivity. The finesse is an important parameter that determines the performance of a Fabry-Perot filter. Conceptually, finesse can be thought of as the number of beams interfering within the Fabry-Perot cavity to form the standing wave. The primary factor that affects finesse is the reflectance R of the Fabry-Perot mirrors, which directly affects the number of beams circulating inside the cavity. The mean= 24.87, median= 10.63, mode= 4.441 and the STD= 42.97. In Figure 5 we found another important factor in the design of the filter is the contrast factor which is defined primarily as the ratio of the maximum to minimum transmission.Figure 6 shows finesse against contrast factor.

Figure 7 represents the relationship between the amplitude transmission and the wavelength. The mean= 0.0003402, median= -0.002776, mode= -0.9718 and the STD= 0.3977. Figure 8shows the power transmission versus the wavelength. The mean= 0.3157, median= 0.1858, mode= 0.0006045 and the STD= 0.3178. Finally, Figure 9 and Figure 10showthe delay and dispersion versus the wavelength after using the transfer function, coupled mode equation andthen POLYFIT function. The theoretically designed delay has a small oscillations around $-0.02106\,ps$ are visible. Of course, the same behavior can be found for the dispersion. The average dispersion is around $-0.001241 ps^2$.The analysis results for the mean, median, mode and the standard deviation STD are tablets in table .2 fordelay and dispersion.

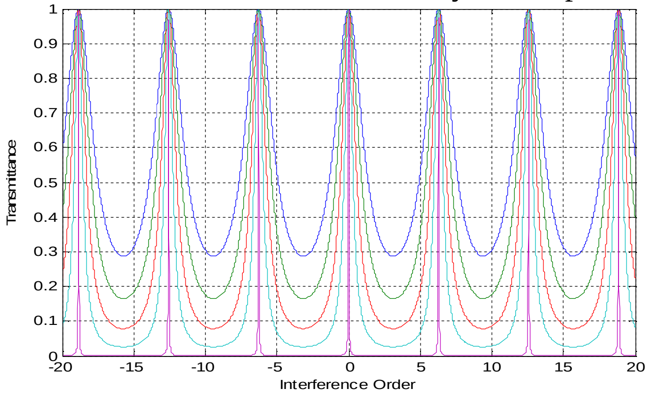


Figure 3: Shows the transmitted intensity versus the interference orderfor various values of transmittance of thecoatings.Not that the peaks get narrower.

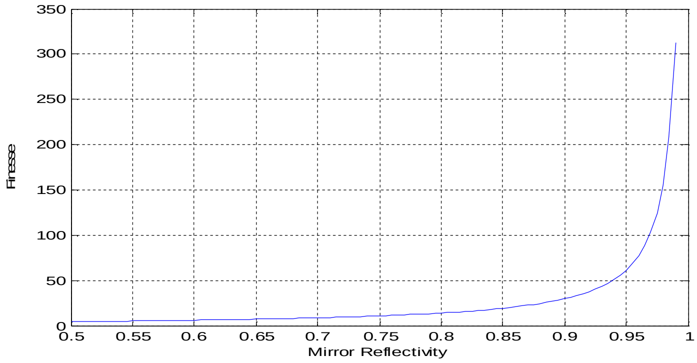


Figure 4: Finesse versus the mirror reflectivity. Not that the coefficient of finesse increases. When peaks are very narrow in Figure 3, light can be transmitted only if the plate separation ℓ, refractive index n, and the wavelength λ satisfy the precise relation: $\delta = 2\pi n\ell \cos\theta / \lambda$.

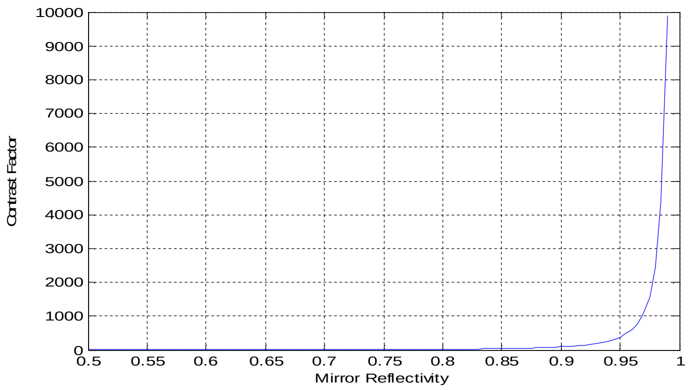


Figure 5: Contrast factorand the mirror reflectivity.The mean= 247.9, median= 11.21, mode= 1.778 and the STD= 1118.

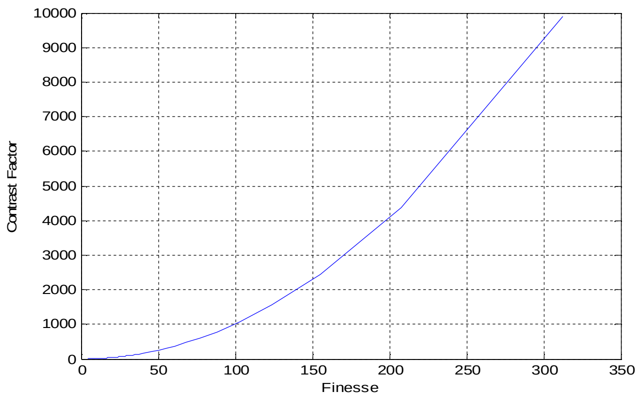


Figure 6: Finesse against contrast factor.Very high finesse factors require highlycontrast factor. These mean, when finesse increase, contrast factor increase also.

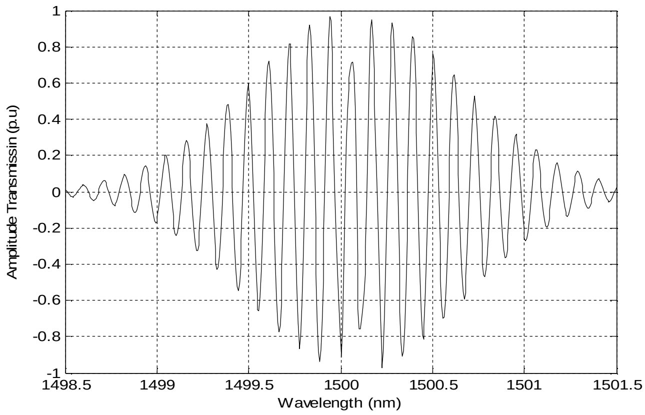


Figure 7: The relationship between the amplitude transmission and the wavelength. The amplitude values are around (-0.9718)-(0.9704).

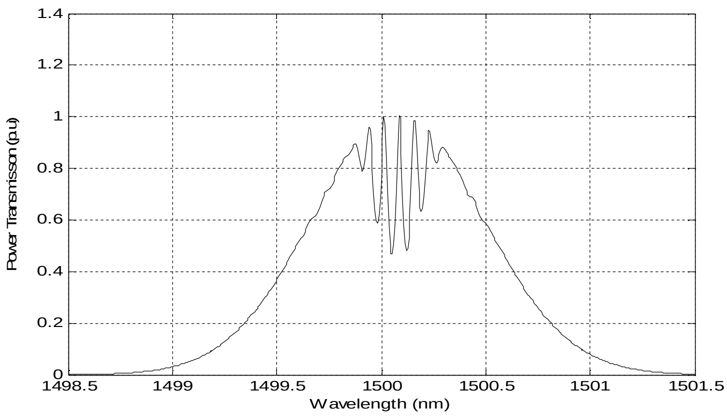


Figure 8: Power transmission versus the wavelength.

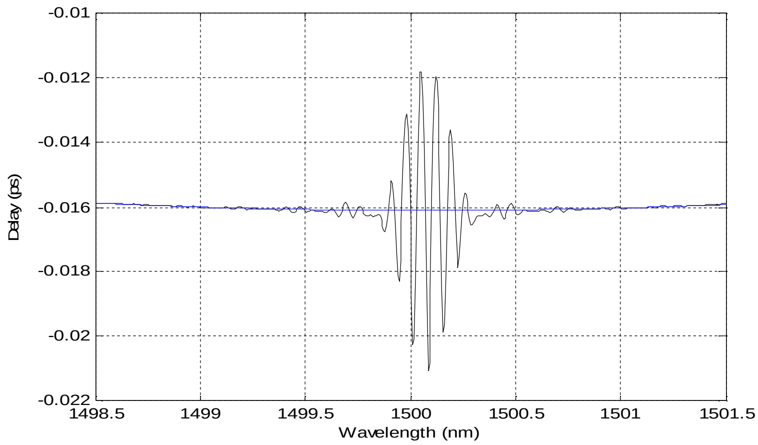


Figure 9: The relationship between the delay and the wavelength. The average fit delay has small oscillationsaround $-0.0161\,ps$.

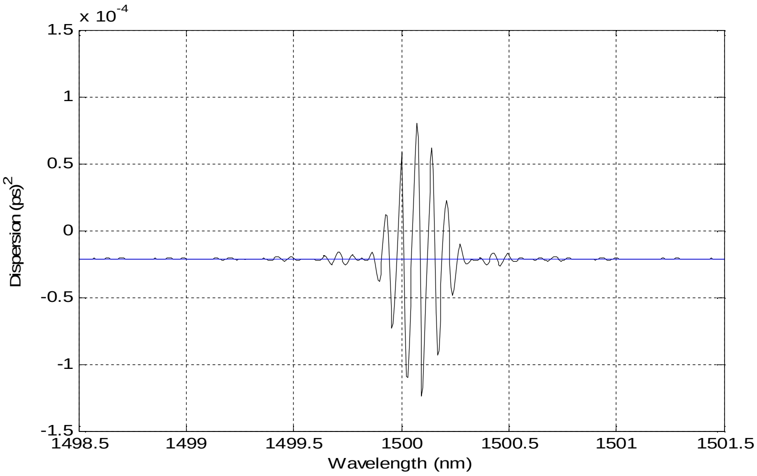


Figure 10: The relationship between the dispersion and the wavelength.The average dispersion is in excellent agreementwith $-0.001241\ ps^2$. Also, the average fit dispersion is in great agreement with $-2.146E-005\ ps^2$.

Table 1: Thestatistical analysis: mean, median, mode and the standard deviation for Fabry-Perot transmittance function.

	Transmittance Function				
	1st	2nd	3rd	4th	5th
Mean	0.5519	0.4237	0.2964	0.1732	0.01103
Median	0.471	0.3041	0.1585	0.05573	0.0002237
Mode	o.2868	0.1648	0.0784	0.02595	0.000101
STD	0.2447	0.2736	0.2769	0.2464	0.07351

Table 2: Thestatistical analysis: mean, median, mode and the standard deviation for the delay and dispersion after and before using POLYFIT function.

	Delay ps	Dispersion ps^2	Fit Delay ps	Fit Dispersion ps^2
Mean	-0.01603	-2.137e-005	-0.01603	-2.137E-005
Median	-0.01602	-2.132e-005	-0.01605	-2.14E-005
Mode	-0.02106	-0.0001241	-0.0161	-2.146E-005
STD	0.000854	1.738e-005	6.181e-005	9.194E-008

Conclusion

This part has presented an intense conclusion on theoretical design of the Fabry-Perot filter. The paper began with a brief historical background. The Fabry-Perot interferometer, simply referred to as the Fabry-Perot, is an important application of multiple wave interference in optics. It consists of two partially reflecting surfaces aligned with each other in such a way that many waves of light derived from the same incident wave can interfere. The resulting interference patterns may be used to analyze the spectral character of the incident beam.This theoretical design study including FSR, finesse and contrast, used to assess the performance of the Fabry-Perot filter were discussed.Low cost practical cavity will always have deviation from the standard analytical model. An attempt is made to analyze the factors that control and affect the performance and the design of the Fabry-Perot filter versus the parameter that control those factors. Very high finesse factors require highly reflective mirrors.A higher finesse value indicates a greater number of interfering beams within the cavity, and hence a more complete interference process. The figure show that the linear increase in finesse with respect to contrast increase. The equation and the plots also show that a linear increase in finesse, translates into a quadratic to each other.The average fit delay and dispersion has small oscillations around the design wavelength.

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