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Symbol Error Probablity of Coherent PSK System in the Presence of Two Path Interference

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Abstract

The effect of two path interference on the symbol -error probability of binary coherent phase shift keyed (CPSK) receiving system is investigated. An expression for the probability of symbol error is derived to evaluate the system performance when a direct strong wanted signal is accompanied with two path interference and Gaussian noise.

Keywords: CPSK, Interference, Multipath fading, Error probability

INTRODUCTION
The presence of interference with the direct signal embedded in additive white Gaussian noise (AWGN) is a common experience of the communication system designer. The interference may appear due to random path length fluctuation in the multipath propagation of information bearing microwave signal. In a binary coherent phase shift keyed (CPSK) receiving system, a carrier tracking loop is commonly used to derive a phase reference from the received signal. For this system two cases may arise regarding phase reference of the receiver:

If the phase fluctuations of the interference are much faster than the time constant of the loop, the reference is in phase with the direct signal. On the other hand,

The reference is in phase with the composite signal (signal + interference) for slow phase variations of the interfering signal.

The effect of general multiple interfering paths ($n>1$, n being number of paths)on the error probability of CPSK system for case (a) has been described in the literature[1,2]. Morgan [1] considered case(b) for single path interference . Lindsey [3] also derived the probability of error for case(b) considering Rician distribution of the received envelope which is however valid for large number of interfering paths. In this article, the effect of two path interference on the error probability of binary CPSK system for case(b) is investigated. Two path interference is separately studied because well-known central limit theorem cannot be successfully applied here and the envelope of the received signal would not be Rician distributed as considered in [1,3]. In the presentanalysis, both the interference and noise powers are assumed small in comparison with the signal power to obtain a close-form expression of the error probability. However these assumptions are appropriate in most real situations.

ANALYSIS

The input $r(t)$to the receiver can be written as the sum of an information bearing signal$S(t)$, interference$I(t)$and AWGN n(t) of power N_0. Therefore,

$$r(t) = S(t) + I(t) + n(t) \tag{1}$$

The information bearing signal $S(t)$is a unity amplitude sinusoid of the form

$$S(t) = \sin[\omega t + \varphi(t)] \tag{1a}$$

Here ω is the angular frequency of the carrier and $\varphi(t)$is the information bearing signal phase. For a binary CPSK signal

$$\varphi(t) = d_n(t).\frac{\pi}{2} \tag{1b}$$

where $d_n(t)$ is the zero mean data stream of value $+1$ or-1.

For two interfering paths $I(t)$ is taken as

$$I(t) = \sum_{i=1}^{2} x_i \sin[\omega t + \varphi(t) + \theta_i(t)] \tag{1c}$$

where x_i^2 and $\theta_i(t), (i = 1,2)$ are the interference to signal power ratio (ISR) and phase of the i^{th}interference. Both the phases $\theta_1$and θ_2 are uniformly distributedrandom variable within the range –π to π and independent of each other. It is assumed that the time rate of fluctuation of $\theta_1$and θ_2 is slow compared to the bit rate and hence the phases are quasi-stationary over the bit period. The joint probability density function (pdf) of $\theta_1$and $\theta_2$is

$$p(\theta_1, \theta_2) = \frac{1}{4\pi^2} \tag{1d}$$

Substitution of (1a) and (1c) into (1) gives

$$r(t) = a.\sin[\omega t + \varphi(t) + \theta(t)] + n(t) \tag{2}$$

where

$$a^2 = \left(1 + \sum_{i=1}^{2} x_i \cos\theta_i\right)^2 + \left(\sum_{i=1}^{2} x_i \sin\theta_i\right)^2 \tag{2a}$$

and

$$\theta = \arctan\frac{\sum_{i=1}^{2} x_i \sin\theta_i}{1+\sum_{i=1}^{2} x_i \cos\theta_i} \tag{2b}$$

If $x_i (i = 1,2) \ll 1$, (2a) can be approximatedby neglecting the product term as

$$a^2(t) \approx (1 + x_1^2 + x_2^2)\left[1 + \frac{2(x_1\cos\theta_1 + x_2\cos\theta_2)}{1+x_1^2+x_2^2}\right] \tag{2c}$$

In case (b) the reference is in phase with the composite signal as illustrated in figure 1and the instantaneous bit energy E at the receiver output is

$$E = a^2(t)\frac{T}{2} \tag{3}$$

Where T is the bit period.

The signal to noise power ratio, SNR(R) at the receiver output is

$$R = \frac{E}{N_0} = \frac{a^2T}{2N_0} = R_0a^2 \tag{4}$$

Where $R_0 = \frac{T}{2N_0}$ is the output SNR in the absence of interference.

Substitution of (2c) into (4) gives

$$R(t) = R_0(1 + x_1^2 + x_2^2)[1 + \frac{2(x_1\cos\theta_1 + x_2\cos\theta_2)}{1 + x_1^2 + x_2^2}] \tag{5}$$

Now the probability of symbol error (P_e)for CPSK system in a background of AWGN is known as [4]

$$Pe = \frac{\mathrm{erfc}\sqrt{R}}{2} \tag{6}$$

where

$$\mathrm{erfc}(y) = \frac{2}{\sqrt{\pi}}\int_y^\infty \exp(-u^2)du \tag{6a}$$

and R is the output SNR of the detector.

For large SNR values, (6) reduces to

$$P_e \approx \frac{e^{-R}}{2\sqrt{\pi R}} \quad (7)$$

After substitution of (5) into (7), P_e simplifies to

$$P_e(x_1, x_2, \theta_1, \theta_2) \approx \frac{e^{-R_0}}{2\sqrt{\pi R_0}} \frac{e^{-R_0\left(x_1^2+x_2^2\right)}}{\sqrt{1+x_1^2+x_2^2}} e^{-2(R_0'x_1'\cos\theta_1 + R_0'x_2'\cos\theta_2)}(1 - x_1'\cos\theta_1 - x_2'\cos\theta_2) \tag{8}$$

where

$$R_0' = R_0(1 + x_1^2 + x_2^2) \tag{8a}$$

and

$$x_i' = \frac{x_i}{1+\sum_{i=1}^{2} x_i^2} \tag{8b}$$

The average value of symbol error probability can be found out from the following averaging rule:

$$P_e(x_1, x_2) = \iint_{-\pi}^{\pi} P_e(x_1, x_2, \theta_1, \theta_2)\, p(\theta_1, \theta_2) d\theta_1\, d\theta_2 \tag{9}$$

Substituting (1.d) and (8) into (9) results in the expression

$$P_e(x_1, x_2) = P_e(0)\frac{e^{-R_0\left(x_1^2+x_2^2\right)}}{\sqrt{1+x_1^2+x_2^2}} I_0(u)I_0(v)\left[1 + x_1'\frac{I_1(u)}{I_0(u)} + x_2'\frac{I_1(v)}{I_0(v)}\right] \tag{10}$$

where

$u = 2R_0x_1$ and $v = 2R_0x_2$.

Here, $P_e(0) = \frac{e^{-R_0}}{2\sqrt{\pi R_0}}$ $(R_0 \gg 1)$ is the probability of symbols error in the absence of interference. The values of $P_e(x_1, x_2)$ are calculated from (10) for different values of R_0 and $x_i(i = 1,2)$ that are valid in the approximation. The calculated values of P_e are plotted in Figure 2 as a function of R_0 for different sets of values of x_1 and x_2. In Figure 3, P_e is plotted as a function of x_1^2(or x_2^2) for definite values of SNR (R_0) and total ISR $(\sum_{i=1}^{2} x_i^2)$.

CONCLUSION

A simple approach is adopted in the analysis to derive an expression for the probability of symbol error of a binary CPSK receiving system when a direct strong signal is corrupted with two path interference in the background of Gaussian noise. The increase of error probability with the increase of interference to signal amplitude ratio (x_i)indicates that the receiver performance is degraded in the presence of multipath interference and the degradation effect is more pronounced when SNR is high. It is also observed that for a fixed SNR value error probability depends on the distribution of interference power in the interfering paths. The worst case arises when total interference power is equally distributed within the paths. Certain realistic approximations are taken in the analysis to simplify the mathematics. However this correspondence presents an analytical tool for evaluating the performance degradation of a CPSK receiver in the presence of interference.

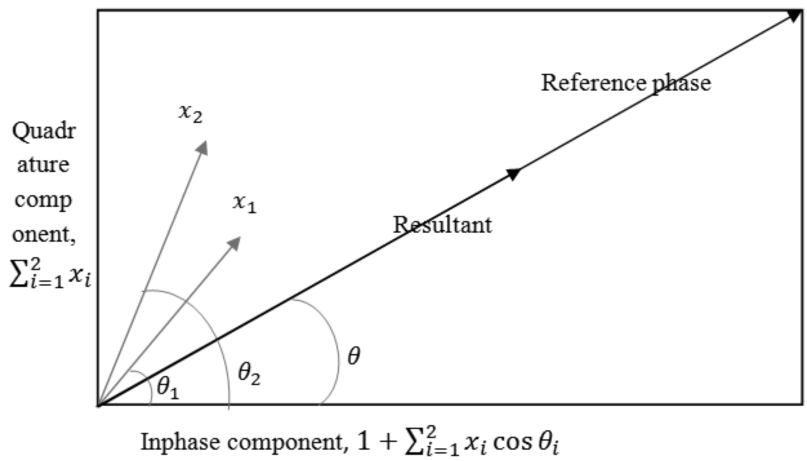

Figure 1: Phasor diagram of composite signal and coherent reference

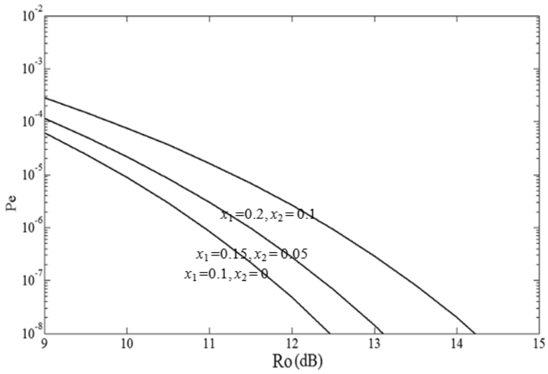

Figure 2: Probability of symbol error vs. SNR

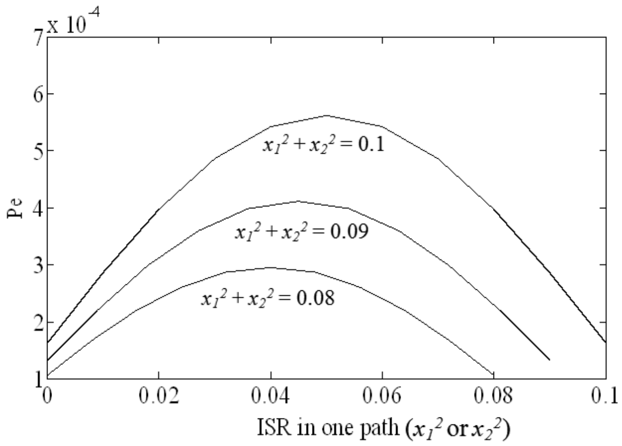

Figure 3: Pe vs. ISR in one path for $R_0 = 10.0$

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