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Similarity of Moment of Inertia

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ABSTRACT

We can have infinite number of arrangement of different object to get infinite number of other objects. Interestingly if we arrange some identical bodies (same mass distribution, shape, size etc) under certain conditions the final body so obtained maintains some order or similarity of rotational inertia (I). This idea is helpful to calculate MI of some 3D bodies considering 2D approach similarly 2D in 1D approach.

ABBREVIATIONS

MI	- moment of inertia
COM	-center of mass

INTRODUCTION

Concept of MI plays crucial role in rotational mechanics. In generally we use method of integration, theorem of perpendicular and parallel axes to calculate MI ; using the basic formula $I=\sum m_i r_i^2$. To define many physical properties like kinetic energy of rotating body, effect of torque, angular momentum etc we need to properly understand concept of MI. We can think of bigger objects to be a special arrangement of some smaller objects. Under certain condition if we arrange some identical bodies then the final body so obtained would have similar calculating formula of MI or it behaves in the same way as the initial identical bodies behave. which led me to propose the following theorem:

THEOREM 1:

The MI about a perpendicular axis of a body formed due to the arrangement of some identical bodies (same size, shape, mass) is similar to MI of each unit body ; if parallel line/lines drawn through all identical particles of each single body is/are at an equal distance from the axis about which MI is to be measured.

Proof: let AOB be a perpendicular axis passing through a body (fig1). Let m_i be the mass of i^{th} particle at a distance of r_i from AOB.

So MI of the body about AOB $=\sum m_i r^2$......... (1)

Let n number such bodies are arranged in such a way that parallel line/lines to AOB drawn through all identical bodies are at same distance from AOB (fig2).

So MI of the final body is= $\sum m_i r_i^2 + \sum m_i r_i^2 + \ldots\ldots + \sum m_i r_i^2$......n times

$= n\sum m_i r_i^2 = \sum (nm_i) r_i^2 = \sum m_i' r_i^2$ where $m'=nm$ .... (2)

From (1) & (2) it is clear that calculating formulae are similar. (This condition of arrangement is unique to maintain similarity. See next theorem)

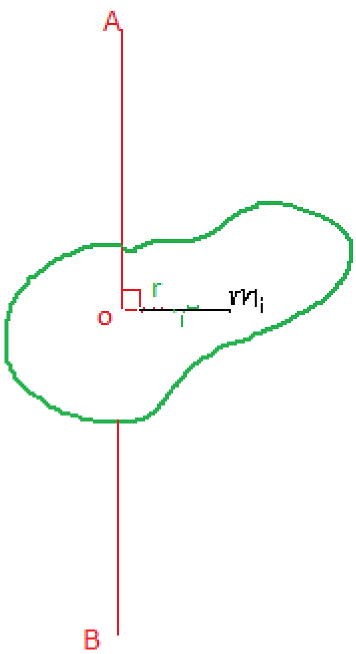

Fig1

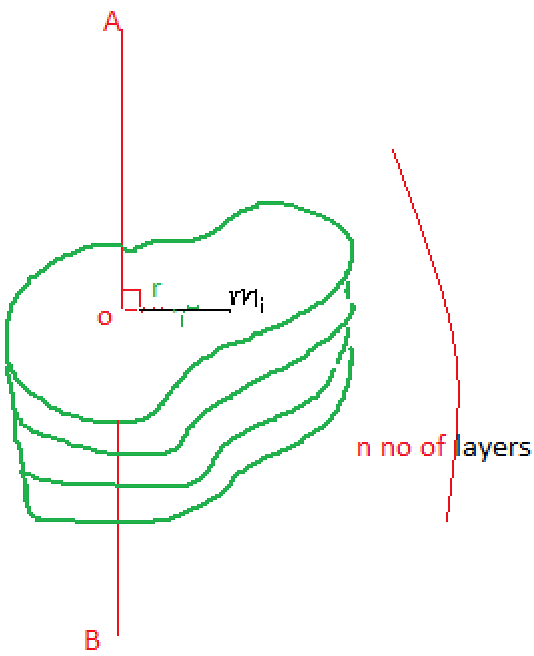

fig2

Theorem 2:

The MI about a perpendicular axis of a body formed due to the arrangement of identical bodies is not similar to each of single unit bodies if parallel line/lines (to perpendicular about which MI is calculating) drawn through all identical particles of all unit bodies are not at the same distance from the perpendicular about which MI is calculated.

Proof:

Since all bodies are identical their position of center of mass (COM) would be at a same position w.r.t to each body. But if parallel line/lines drawn through all COM of all unit bodies after arrangement are not at an equal distance from the perpendicular through the body then their radius of gyration would be different leading to non – similarity of MI of final body with all its unit bodies.

DISCUSSION:

Observe the table 1 in which formulae for both the pairs is similar. Similarly we can have many example where we can find such similarity. Theorem1 strongly predict the formula to calculate MI of objects that would formed after applying such conditions. consider pair 1; a uniform hollow cylinder can be thought of as an arrangement of ring ; similarly a uniform solid cylinder can be thought of as an arrangement of uniform circular plate. Hence calculating formula once but using for two different objects. Not only this arrangement but infinite numbers of such arrangement possible which would maintain this similarity provided following the conditions of theorem 2.

TABLE1:

PAIRS	OBJECTS	FORMULA	REMARKS
1	Uniform ring of radius r and mass m	$I=mr^2$	MI about perpendicular axis through center
	Uniform hollow cylinder of radius r and mass m	$I=mr^2$	MI about perpendicular axis through center
2	Uniform circular plate of radius r	$I_z=mr^2/2$	MI about perpendicular axis through center
	Uniform solid cylinder of radius r	$I_z=mr^2/2$	MI about perpendicular axis through center

CONCLUSION:

This similarity approach is very helpful it determines the limit of extension of arrangement which would maintain similarity of the calculating formulae. Here only about arrangement of identical bodies is discussed because if we consider non - identical bodies it would be like adding different numbers and finding the sum to be integral multiple of each numbers involved.