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Instability of Cold Beam in Dense Hot Plasma

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Abstract

The author investigate the effect of cold electron beam upon the process of nonlinear interactions of waves in hot plasma. It has been shown that the nonlinear effects is not enough for description of instability saturation that appear at interaction of cold beam of electrons with hot plasma. Quite possible that instability may be obtained if due to quasi-linear effects and inhomogeneity of plasma.

Introduction

A great number of publications /1-4/, and ref. cited therein, are devoted to the theoretical study beam-plasma instability. The excitation of waves incident on plasma has been investigated by many authors /5-7/. They considered the case when the oscillation frequency ω is less than the Langmuir frequency of the plasma ω_p. In the present work, we studied the kinetic instability in a hot non-magnetized plasma by a cold beam of electron with low density $n_1 < < n_0$ (n_1 – is the density of the beam). Beam velocity $\vec{u}$ may be assumed small compared to thermal speed of electron in plasma ($u << \upsilon_{Te}$).

The instability saturation may be obtained if due regard is given to quasi-linear effects and inhomogeneity of plasma (the latter is important for stabilization of instabilities in the area of longer waves).

Basic equation

Let us consider the excitation of kinetic instability in a hot non-magnetized plasma by a cold beam of electron with a low density $n_1 < < n_0$ (n_1-is density of beam). Beam velocity $\vec{U}$ may be assumed small compared to thermal speed of electrons in plasma ($u << \upsilon_{Te}$). This purely electron instability was analyzed for the first time in ref. /8/. Dispersion equation of linear theory for such oscillation in the following:

$$\varepsilon\left(\bar{k},\omega\right)\equiv 1+\frac{1}{k^{2}d_{\circ}^{2}}\left(1+\frac{i\sqrt{\pi}\omega}{\sqrt{2}k\upsilon_{Te}}\right)-\frac{\alpha\omega_{pe}^{2}}{\left(\omega-\bar{k}\bar{u}\right)^{2}}\text{.........(1)}$$

Where:

$$d_{\circ}^{2}\equiv \upsilon_{Te}/\omega_{pe}$$

$$\alpha\equiv\left(n_{1}/n_{\circ}\right)\ll 1,\omega\ll k\upsilon_{Te}$$

From (1) and according /8/ we may find :

$$\omega_{k}=\bar{k}\bar{u}-\alpha^{1/2}\omega_{pe}\left(1+k^{2}d_{\circ}^{2}\right)^{1/2}.kd_{\circ}.sign\left(\bar{k}\bar{u}\right)\text{........(2)}$$

$$\gamma_{k}'=\left(\frac{\pi\alpha}{8}\right)^{1/2}\left(1+k^{2}d_{\circ}^{2}\right)^{-3/2}.\left\{\left|\bar{k}\bar{u}\right|-k\upsilon_{Te}\alpha^{1/2}\left(1+k^{2}d_{\circ}^{2}\right)^{-1/2}\right\}\text{......(3)}$$

Swaying of oscillations occurs at:

$\upsilon\succ\upsilon_{r}$ where,

$$\upsilon_{r}\equiv\frac{\upsilon_{Te}}{\left|\cos\upsilon\right|}\sqrt{\alpha}\left(1+k^{2}d_{\circ}^{2}\right)^{-1/2}$$

$$\cos\upsilon\equiv\frac{\bar{k}\bar{u}}{ku}\text{..........(4)}$$

Growth increment (4) is at its maximum if $\cos\upsilon=1,kd_{\circ}\ll 1$.

For instance at $\upsilon\gg\upsilon_{r}$, maximum point is determined by meanings

$$\cos\upsilon=1,kd_{\circ}=\frac{1}{\sqrt{2}}\text{.....(5)}$$

And increment in this point equal to :

$$\max\gamma_{k}\equiv\gamma_{m}=\frac{1}{3}\sqrt{\frac{\pi}{6}}\sqrt{\alpha}\,\frac{\upsilon}{\upsilon_{Te}}\omega_{pe}\text{.........(6)}$$

Kinetic instability of the plasma

Now we may take excitation of oscillation in nonlinear approximation, using kinetic equation for waves and matrix elements in ref. /9/ and from relation (30-35) it is not difficult to see that equation (18) in ref. /9/ taken such form:

$$\frac{\partial I_{\bar{K}}}{\partial t}=2I_{\bar{K}}\left\{\gamma_{\bar{k}}+\left(\frac{\partial\xi}{\partial\omega}\right)^{-1}_{\bar{k}\,\omega_{k}}.\int d\bar{k}'I_{\bar{k}'}.Jm\left\langle u^{(e)}_{\bar{k},\bar{k}'}\right\rangle_{\circ}\right\}\text{..........(7)}$$

Excitation of narrow packet of waves occurs, on linear stage in which wave vectors correspond to maximum meaning of increment γ_{k} $\left(k\approx k'\approx k_{m}\right)$ using the condition of packet narrowness $\left(\left|k''\right|\ll k\right)$, then equation (7) will take the form:

$$\frac{1}{2I_{\bar{k}}}\frac{\partial I_{\bar{k}}}{\partial t}=\gamma_{\bar{k}}-\Delta\gamma_k \text{..........(8)}$$

Where:

$$\Delta\gamma_k=\frac{2\pi\sqrt{\alpha}e^2\omega_{pe}^2\omega_k d_\circ^3 k^{2/3}}{m_e^2\upsilon_{Te}^5\wp_e^{2/3}\left(1+k^2 d_\circ\right)^{3/2}}.\left\{a_1\int_0^k dk'k'^2 I_{\bar{k}'}+a_2\int_k^\infty dk'k'^2 I_{\bar{k}'}\right\}\text{.....(9)}$$

$$a_1\equiv\sqrt{\frac{\pi}{2}}\left(\frac{3}{2}\right)^{5/3}\int_0^\infty d\xi.\xi^4 e^{-\xi^2}\int_0^\infty d\tau.e^{-\tau}\left(\tau+\frac{3}{2}\xi^3\right)^{-2}\text{.........(10)}$$

$$a_2\equiv\sqrt{\frac{\pi}{2}}\left(\frac{3}{2}\right)^{5/3}\int_0^\infty d\xi.\xi^4 e^{-\xi^2}\int_0^\infty d\tau.e^{-\tau}\left[\left(\tau+\frac{3}{2}\xi^3\right)^{-2}+\frac{9}{4}\xi^6\right]^{-1}\text{........(11)}$$

Oscillation intensity when nonlinear effects become significant may be evaluated by letting $kd_\circ \ll 1$. In this case, at $\gamma_{\bar{k}}=\Delta\gamma_k$ we have :

$$\zeta\equiv\frac{1}{8\pi}\int d\bar{k}k^2 I_{\bar{k}}\sim m_e\upsilon_{Te}^2 n_\circ\left(\frac{\nu_{ee}}{\omega_{pe}}\right)^{2/3}\text{......(12)}$$

ν_{ee} -Frequency of pair coulomb collisions 9 .

Assume that in a moment of time t value $I_{\bar{k}}(t)$ makes major increment into sub integral expression at $\bar{k}\sim k_\circ(t)$, then for $k\ll\bar{k}$, and according to ref. 9 it s result that :

$$\left\langle u_{\bar{k},\bar{k}'}^{(e)}\right\rangle=\frac{-i\sqrt{\pi}3^{2/3}e^2\omega_{pe}^2 Z_{\aleph}k'}{m_e^2\upsilon_{Te}^4\wp_e^{2/3}k^{7/3}}.\int_o^\infty d\xi.\xi e^{-\xi^3}.\int d\xi'\frac{\xi'}{(\xi+\xi')}e^{-\xi^3}+i\frac{\sqrt{\pi}}{4}\frac{3^{2/3}e^2\omega_{pe}^2 Z_{\aleph'}}{m_e^2\upsilon_{Te}^4\wp_e^{2/3}k^{4/3}}.\int_o^\infty d\xi.\xi e^{-\xi^3}\text{...(13)}$$

As $Z_{\aleph}\sim Z_{\aleph'}$, the second term in (13) is Less than the first one by $\left(k'/k\right)$ times.

Therefore, for small values of $k\ll d_\circ^{-1}$, equation (7) taken the form:

$$\frac{\partial I_{\bar{k}}}{\partial t}=I_{\bar{k}}\sqrt{\frac{\pi\alpha}{2}}\left(\left|\bar{k}\bar{u}\right|-k\upsilon_{Te}\sqrt{\alpha}\right).\left\{1+\frac{a_3 e^2}{m_e^2\upsilon_{Te}^2\wp_e^{2/3}k^{2/3}}\int_0^\infty dk'k'^3 I_{\bar{k}'}\right\}\text{.........(14)}$$

where:

$$a_3\equiv 3^{2/3}.\int_o^\infty d\xi.\xi e^{-\xi^3}\int_o^\infty d\xi'.\xi'\frac{\exp\left(-\xi'^3\right)}{\left(\xi+\xi'\right)^2}\sim 1\text{.......(15)}$$

For $k\succ k'$ and from equation (36) in ref. 9 we shall have :

$$\left\langle u_{\bar{k},\bar{k}'}^{(e)} \right\rangle = \frac{i\sqrt{\frac{\pi}{2}}e^2\omega_{pe}^2 k'^{2/3}\omega_{k'}a_4}{m_e^2\upsilon_{Te}^5\wp_e^{2/3}k^3} \text{......(16)}$$

Where :

$$a_4 \equiv 3^{2/3}.\int_o^\infty d\xi.\xi\int_o^\infty d\xi'\left(\xi+\xi'\right)^{-1}\exp\left[-\left(\xi+\xi'^3\right)\right] \square 1\text{.......(17)}$$

Replacing (16) into (7), we obtain that :

$$\frac{\partial I_{K'}}{\partial t} = 2I_{k'}\left\{\gamma_{k'} - a_4\sqrt{\frac{\pi\alpha}{8}}.\frac{e^2 4\pi}{m_e^2\upsilon_{Te}^2\wp_e^{2/3}\left(1+k^2d_\circ^2\right)^{3/2}}.\int dk\,k'^{8/3}\left|\omega_{k'}\right|I_{\bar{k}'}\right\}\text{.......(18)}$$

The second term in figure brackets (18) is negative. This fact indicates instability attenuation. There with, this second term decrease quicker with the increase in K than the first one, so that no instability saturation results (at $kd_\circ \square 1$).

Comparing the behavior of oscillation interesting with time in the area of large and small K, one may conclude that there occurs the so called nonlinear transfer of waves by the spectrum in to the area of small K. (similar effects is observed at nonlinear attenuation of lengmure wave/10, 11/).

Therefore taking in to account nonlinear effects is not enough for description of instability saturation that appear at interaction of cold beam of electrons with hot plasma. Quite possible that instability saturation may be obtained if due regard is given to quasi-linear effects and inhomogeneity of plasma (the latter is important for stabilization of instabilities in the area of longer waves)

Conclusion

The author show that coulomb collision influence greatly the nonlinear dispersion of waves during interaction of cold beam of electrons with free hot plasma. The use of kinetic wave equation of homogenous plasma alone in this instance does not allow to describe the process of complete saturation of instability due to nonlinear transfer of waves along the spectrum in to the area of small k, where discharge of wave energy is not taken into consideration. Therefore, nonlinear effects is not enough for description of instability saturation that appear at interaction of cold beam of electrons with hot plasma.

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