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Beam Interaction With A Bounded Plasma

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Abstract

Application of waves by the electron beam in an inhomogeneous bounded plasma is investigated. It is shown that the energy of electrostatic oscillations amplified by the beam in a plasma is effectively absorbed by the plasma at the outlet of a beam. The investigation is fulfilled in the approximation when the width of plasma-vacuum transition region is small in comparison with the wavelengths.

Introduction

Recently there has been considerable interest in the problem of the collective interaction of charged particle beam with plasma /1-5/. Such interactions arise from a coupling of perturbations in the macroscopic density and current of the beam with those in the plasma through the associated electromagnetic field. A great interest for development of new method in amplification and generation of electromagnetic waves /6,7/, acceleration of charged particles in plasma /8/, high frequency heating of plasma /9/ and so on.

In the present work, we considered a homogeneous plane-parallel plasma layer with a narrow (as compared with the wavelength) in homogeneous plasma – vacuum transition regionwaves are considered as longitudinal, monochromatic oscillations directed along the propagating beam(along the x-axis).

Basic Equations

The physical situations to which this theoretical investigation is intended to be most applicable is that of an externally generated electron beam that is injected in a plasma. The plasma is characterized by $\varepsilon(x)=1-\frac{\omega_p^2}{\omega(\omega+i\upsilon)}$, where $\omega_p(x)$ is the electron plasma Langmuir frequency and << ω is the collision one (is neglected everywhere except at R_e ε ≈ 0). Let us consider a beam with an average equilibrium density n_0 moving with a velocity $\upsilon_\circ$ along X- axis. Such a situation is described in the following manner.

$$N = n_\circ + n$$

$$\bar{V} = \vec{e}_x \upsilon_\circ + \vec{\upsilon}$$

The quantities n and □ give the amplitudes of the fluctuating density and velocity components, both assumed very small compared with the corresponding average quantities. These quantities have a sinusoidal variation in time given by $e^{-i\omega t}$

The continuity equation:

$$\frac{\partial N}{\partial t} + \bar{\nabla}.\left(N\vec{V}\right) = 0 \text{.........(1)}$$

And the equation of motion :

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}.\nabla\right)\vec{V} = \frac{e}{m}\vec{E} \text{.........(2)}$$

Where $\vec{E}$ is the electric field on longitudinal coordinate x , transverse direction y , and time t can be formulated as :

$$\vec{E}\left(r,t\right) = \vec{E}\left(x,y\right)e^{-i\omega t} + c.c.$$

e , m are the charge and mass of the electron .

As a result equations (1) and (2) reduce to :

$$n = -\left(\frac{n_\circ}{\upsilon_\circ}\right)e^{i\aleph x}\int_0^x \left(\vec{\nabla}.\vec{\upsilon}\right)e^{-i\aleph X'}dX' \text{.........(3)}$$

$$\upsilon = \frac{e}{m\upsilon_\circ}e^{i\aleph x}\int_0^x \vec{E}e^{-i\aleph X'}dX' \text{.........(4)}$$

Where $\aleph = \dfrac{\omega}{\upsilon_\circ}$

ω is frequency of external alternating voltage .

The electric field is related to the density by Poisson,s equation

$$\bar{\nabla}.\left(\varepsilon\bar{E}\right) = 4\pi e n \text{.........(5)}$$

Substituting the value of n given by equation (3) into Poisson,s equation (5) , one obtains :

$$\bar{\nabla}.\left\{\frac{\partial^2}{\partial X^2}\left(\varepsilon\bar{E}\right) - 2i\aleph\frac{\partial}{\partial x}\left(\varepsilon\bar{E}\right) - \aleph^2\left(\varepsilon - \frac{\omega_p^2}{\omega^2}\right)\vec{E}\right\} = 0 \text{.........(6)}$$

For a one-dimensional case $\left(\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0\right)$ equation (6) reduces to

$$\frac{d^2E}{dx^2} + \frac{k^2}{\varepsilon} E = 0 \text{..........(7)}$$

Where : $k = \frac{\omega_b}{\upsilon_\circ}$ and $E = \varepsilon E_x e^{-i\aleph x}$

In the regions of the plasma homogeneity we can write the solutions of equation (7) in the form :

$$E = A_p e^{ikx} + Be^{-ikx} \text{ at } x \le 0 \text{.........(8)}$$

$$E_p = A_p e^{k(x-a)/\sqrt{\mu}} + B_p e^{-k(x-a)/\sqrt{\mu}} \text{ at } a \le x \le l \text{.........(9)}$$

Where A, B, A_P, B_P are the integration constants ,

$\varepsilon(x) = 1$ at $x \le 0, x \ge l + b$

$\varepsilon(x) = -\mu$ at $a \prec x \prec l$

Where :

$$\mu \succ 0, \frac{\partial \mu}{\partial x} = 0 \text{.....(10)}$$

$ka \ll 1, kb \ll 1$

Within the inhomogeneous region we shall solve equation (7) only for waves whose plasma-free wavelength is much greater than the characteristic length of the inhomogeneity of the plasma density, I .e., we shall suppose the inequalities $ka \ll 1$ and $kb \ll 1$ to hold. Then the method of successive approximation can be used and a solution can be found in the form:

$$E = E^{(0)} + E^{(1)} + \text{............(11)}$$

Where $E^{(0)}$, $E^{(1)}$, …….. satisfy the differentialequation.

$$\frac{d^2E^{(0)}}{dx^2} = 0 \text{........(12)}$$

$$\frac{d^2E^{(n)}}{dx^2} + \frac{k}{\varepsilon} E^{(n-1)} = 0$$

In the zero approximation we get :

$$E^{(0)} = c_0 x + \wp_0 \text{ at } 0 \le x \le a \text{........(13)}$$

$$E^{(0)} = c(x - l) + \wp \text{ at } l \le x \le l + b \text{........(14)}$$

Where $c_{0,}\wp_0, c, \wp$ are integration constants .

Matching the field expression at X = 0, a, l and $l+b$ where the function E_x and $\frac{d\ E}{dx}$ one continuous, we can obtain the system of equations:

$$\left.\begin{aligned}
&A+B=D_\circ\\
&ikA-ikB=C_\circ\\
&C_\circ a+D_\circ=A_p-B_p\\
&C_\circ=\frac{k}{\sqrt{\mu}}A_p-\frac{k}{\sqrt{\mu}}B_p\\
&D=A_P e^{\frac{k}{\sqrt{\mu}}(l-a)}+B_p e^{\frac{k}{\sqrt{\mu}}(l-a)}\\
&C=\frac{k}{\sqrt{\mu}}A_P e^{\frac{k}{\sqrt{\mu}}(l-a)}-B_p e^{-\frac{k}{\sqrt{\mu}}(l-a)}.\frac{k}{\sqrt{\mu}}
\end{aligned}\right\} \qquad (15)$$

From (15) we can obtain all integration constants in terms of the amplitude A and B. As a result, neglecting terms of the order of k_a, k_b and $\exp\frac{-kl}{\sqrt{\mu}}, \exp\frac{kl}{\sqrt{\mu}}$ □ $\frac{ka}{\sqrt{\mu}}, \frac{kb}{\sqrt{\mu}}$, we get

$$D=\frac{1}{2}.\exp\frac{kl}{\sqrt{\mu}}\left[\left(1+i\sqrt{\mu}\right)A+\left(1-i\sqrt{\mu}\right)B\right]......(16)$$

The exponent in the right hand side of (16) indicates an order of wave amplification by the beam.

The amount of energy absorbed by the plasma:

We can calculate the amount of energy absorbed by the plasma as a result of wave dissipation. In the homogeneous region (a≤ X ≤ l) the dissipation can be neglected due to smallness of □. Energy absorption at the input of the beam into the plasma can be neglected in comparison with that at the output where the field is exponentially larger. As the result the absorption remains to be calculated in the interval $l \prec x \prec l+b$.

Absorbing of energy S per unit time by the plasma is determined by the work of the field on the electric current in the plasma.

$$S=\int dx j_x E_x=\frac{\omega}{2\pi}\int dx\,.\eta m\varepsilon\frac{|E|^2}{|\varepsilon|^2}.......(17)$$

Where $j_x=\frac{i\omega}{4\pi}\left(1-\varepsilon\right)E_x$ Is the density of electric current produced in the plasma electrons.

Due to the smallness of $\eta m\varepsilon$, in (17) integration proves to be essential only in the small region of the interval $(l, l+b)$, where $R_e\varepsilon$ is close to zero. In this region the plasma's dielectric permeability can be written in the form :

$$\varepsilon(x) = \frac{x - x_\circ}{h} ti\,\delta ....(18)$$

Where :

$$\delta = \frac{\nu}{\omega} \square\ 1, h^{-1} = \left.\frac{\partial R_e\varepsilon}{\partial x}\right|_{x=x_\circ} \square\ b$$

According to (14) , in the region $l \le x \le l+b$ we have :

$$E \approx E^{(\circ)} \approx D ....(19)$$

Using (19) and (18) in (17) and integrating out , we obtain :

$$S = \frac{\omega h}{2}|D|^2 ....(20)$$

If A or B equal zero , relation (20) can be written by means of (16) in the form :

$$S = \frac{\pi}{2}(1+\mu)\exp^{2kl/\sqrt{\mu}} .\omega h W_E ....(21)$$

Where $W_E = \frac{|A|^2}{4\pi}$ or $\frac{|B|^2}{4\pi}$ Is the energy density of electrostatic field in the region X < 0.

From relation (21) it can be readily seen that the energy of the electrostatic oscillations amplified as a result of the passage of the beam through the plasma, is effectively absorbed by the plasma at the outlet of the beam. The value of the absorbed power by the plasma is proportional to the width of transitional plasma-vacuum region h~ b. The magnitude of S (h) will be maximum at h ~ 1/k , but due to invalidity of conditions (10), relation (21) in this case holds true only by an order of magnitude. In case of further increase of h up to the magnitude kh>> 1, power absorbed by the plasma does not increase and is independent of h.

Conclusion

We considered a homogenous plane- parallel plasma layer with a narrow inhomogeneous plasma- vacuum transition region, waves are considered as longitudinal monochromatic oscillations directed along the propagation beam (along the X-axis).

When the width of plasma- vacuum transition region in comparison with the wave lengths, the energy density of electrostatic field amplifies as a result of the passage of the beam through the plasma.

It is effectively absorbed by the plasma at the outlet of the beam. The value of the absorbed power by the plasma is proportional to the width of transitional plasma-vacuum region.

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