

Five Dimensional String Cosmological Models In Barber's Second Self Creation Theory

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Abstract

In this paper we have investigated five dimensional String Cosmological Models with bulk viscosity, Barber's second self creation theory and determine it's solution for three cases. The various physical and kinematical properties are also studied.

Keywords: Bulk viscosity, cosmic string, five dimensional cosmological models, Barbers second self creation theory.

1.Introduction

Several modification of Einstien's general relativity have been proposed and extensively studied so far by many cosmologists to unify gravitation and many other effects in the universe. Barber [1] has produced two continuous self creation theories by modifying the Brans-Dicke theory and general relativity. The modified theories create the universe out of self contained gravitational and matter fields. Barber has included continues creation of matter in these theories. The universe is seen to be created out of self contained gravitational scalar and matter fields.

Brans [2] has pointed out that Barber's first theory is not only in disagreement will experiment, as well as inconsistent, since the equivalence principle is violated. Barber's second theory is a modification of general relativity to a variable G-theory. The second theory retains the attractive features of the first theory and overcomes previous objections. In this theory the scalar field does not directly gravitate, but

scalar couples to the trace of energy momentum tensor. This theory predicts the same precession of the perihelion of the planets on general relativity and in that respect agrees with observation to within one percentage. In the limit, the theory approaches the standard relativity in every respect.

Pimental [3] and Soleng [4] have presented the Robertson Walker solutions of Barber's second self creation theory of gravitation by using power law relation between the expansion factor of the universe and the scalar field Soleng [5], Sing [6], Reddy [7,8,9], Reddy et al. [10,11] and Maharaj and Beesham [12] are some of the authors who have studied various aspects of the two self creation theories. Venkateswarlu and Reddy [13] presented Bianchi type-V radiating model in Barber's first theory. While Venkateswarlu and Reddy [14] have obtained an anisotropic cosmological model in this theory. Reddy and Venkateswarlu [15,16] have obtained spatially homogeneous and anisotropic Binachi type-VI cosmological models in Barber's second self creation theory of gravitation both in vacuum and in presence of perfect fluid with pressure equal to energy density.

Rao and Sanyasi Raju [17,18] have discussed Binachi type-VIII and IX in zero mass scalar fields and self creation cosmology. Shanti and Rao [19] obtained Binachi type-II and III models in self creation cosmology. Micro and Macro cosmological model in Barber's second self creation theory have been studied by Mohanty et al. [20]. Venkateswarlu et al. [21] have studied Binachi type-I,II,VIII and IX string cosmological solutions in self creation theory of gravitation. Rao et al. [22] have discussed exact Binachi type-II,VIII and IX string cosmological models in general relativity and self creation theory of gravitation. In this paper we investigated Binachi type-I cosmological model in Barber's second self creation theory in five dimensional space time.

2. The Field Equations

We consider the five dimensional line elements in the form

$$ds^2 = -A^2(dx^2 + dy^2 + dz^2) - B^2dm^2 + dt^2 \quad (1)$$

Where A and B are functions of cosmic time t only.

The energy momentum tensor for a cloud string dust with a bulk viscous fluid is given by Letelier (1979), Landau and Lifshitz (1963) as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi u^k{}_{;k} (g_{ij} + u_i u_j) \quad (2)$$

$$u_i u^j = -1, \quad x_i x^j = 1, \quad \text{and} \quad u^i x_j = 0 \quad (3)$$

We assume here that the commoving coordinates

$$u^0 = u^1 = u^2 = u^3 = 0, \quad u^4 = 1 \quad (4)$$

The field equations based on Lyra's manifold as proposed by Sen [23] and Sen and Dunn [24] are

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\varphi_k\varphi^k = -8\pi\Phi^{-1}T_{ij} \quad (5)$$

$$\square\Phi = \frac{8}{3}\pi\eta T \quad (6)$$

$$\text{Where } \varphi_i \text{ is the displacement vector given by } \varphi_i = (0,0,0,0,\beta) \quad (7)$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\ddot{A}\dot{B}}{AB} + 2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{3}{4}\beta^2 = -8\pi\Phi^{-1}(\xi\theta + \lambda) \quad (8)$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\ddot{A}\dot{B}}{AB} + 2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{3}{4}\beta^2 = -8\pi\Phi^{-1}\xi\theta \quad (9)$$

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\ddot{A}}{A} + \frac{3}{4}\beta^2 = -8\pi\Phi^{-1}\xi\theta \quad (10)$$

$$-3\frac{\dot{A}^2}{A^2} - 3\frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = 8\pi\Phi^{-1}\rho \quad (11)$$

Where the . Denotes ordinary differentiation with respect to t.

$$\ddot{\Phi} + \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{\Phi} = \frac{8}{3}\pi\eta(4\xi\theta + \lambda + \rho) \quad (12)$$

3. Cosmological Solution

Here we have five independent field equations in seven unknowns. Hence in order to get the solution let us take

$$B=g(A(t)) \quad (13)$$

From equation (9) and (10) we get

$$\dot{A} \left[\frac{1}{A} - \frac{1}{g} \frac{dg}{dA} \right] + \dot{A}^2 \left[-\frac{1}{g} \frac{d^2g}{dA^2} + \frac{2}{A^2} - \frac{1}{2Ag} \frac{dg}{dA} \right] = 0 \quad (14)$$

Equation (14) is satisfied for the following cases

$$\text{Case 1 } \dot{A} = 0 \quad (15)$$

$$\text{Case 2 } \ddot{A} = 0 \quad \left[\frac{1}{g} \frac{d^2g}{dA^2} - \frac{2}{A^2} + \frac{1}{2Ag} \frac{dg}{dA} \right] = 0 \quad (16)$$

$$\text{Case 3 } \left[\frac{1}{A} - \frac{1}{g} \frac{dg}{dA} \right] = 0 \quad \text{and} \quad \left[\frac{1}{g} \frac{d^2g}{dA^2} - \frac{2}{A^2} + \frac{1}{2Ag} \frac{dg}{dA} \right] = 0 \quad (17)$$

CASE-1:

$$\dot{A} = 0$$

$$A = C_1 \quad B = g(C_1)$$

Where C_1 is the integrating constant.

Hence five dimensional string models in this case, after using transformations, reduces to the form

$$ds^2 = -C(dx^2 + dy^2 + dz^2) - Ddm^2 + dt^2 \quad (18)$$

Where $C = C_1^2$ $D = B^2 = (g(C_1))^2$ are constant.

$$\lambda = 0 \quad \theta = 0$$

$$\phi(t) = K_1 e^{\sqrt{\eta} \frac{\rho}{2}} + K_2 e^{-\sqrt{\eta} \frac{\rho}{2}}$$

$$\rho = \frac{3\beta^2}{32\pi} \phi$$

$$\xi = -\frac{3\beta^2}{32\pi\theta} \phi$$

Since $\theta = 0$ therefore we conclude that the Lyra geometry and cosmic strings do not survive in this case and space time becomes Minkopwaskian.

CASE-2:

$$\ddot{A} = 0 \quad \left[\frac{1}{g} \frac{d^2 g}{dA^2} - \frac{2}{A^2} + \frac{1}{2Ag} \frac{dg}{dA} \right] = 0$$

$$A = K_3 t + K_4$$

Where $K_3 \neq 0$ is a constant and K_4 is the integrating constant.

$$B = K_5 A^{-2} + K_6 A$$

Hence five dimensional string models in this case

$$ds^2 = -(K_3 t + K_4)^2 (dx^2 + dy^2 + dz^2) - [K_5 (K_3 t + K_4)^{-2} + K_6 (K_3 t + K_4)]^2 dm^2 + dt^2 \quad (19)$$

$$\xi\theta + \rho = 0$$

$$\theta = \frac{3A}{A} + \frac{\dot{B}}{B} = \frac{3K_3}{K_3 t + K_4} + K_3 \left[\frac{-2K_5 (K_3 t + K_4)^{-3} + K_6}{K_5 (K_3 t + K_4)^{-2} + K_6 (K_3 t + K_4)} \right]$$

$$\rho = -\xi\theta$$

$$\lambda = 0$$

$$\phi(t) = Kt - \left[\ln \left(\frac{A}{1+K_7 A} \right)^{1/3} + \frac{5}{3\sqrt{3}K_7} \tan^{-1} \left(\frac{2K_7 A + 1}{\sqrt{3}} \right) - \frac{1}{6K_7} \ln \left\{ \left(K_7 A + \frac{1}{2} \right)^2 + \left(\frac{\sqrt{3}}{2} \right)^2 \right\} \right]$$

K_7 is the integrating constant.

CASE-3:

$$\left[\frac{1}{g} \frac{dg}{dA} - \frac{1}{A} \right] = 0 \quad \text{and} \quad \left[\frac{1}{g} \frac{d^2 g}{dA^2} - \frac{2}{A^2} + \frac{1}{2Ag} \frac{dg}{dA} \right] = 0$$

$$B = K_7 A + K_8$$

In particular let us take $K_8 = 0$

Take $A = \sin(at + b)$

Where $a \neq 0, b$ are constant.

$$B = K_7 \sin(at + b)$$

(K_8 is the integrating constant) are depends upon A.

Hence five dimensional strings model is in this case

$$ds^2 = -(\sin(at + b))^2(dx^2 + dy^2 + dz^2) - (K_7 \sin(at + b))^2 dm^2 + dt^2 \quad (20)$$

For all t the model is flat five dimensional space times.

If $A = e^{(at+b)}$, Where $a \neq 0, b$ are constant, then $B = K_7 e^{(at+b)}$

Hence five dimensional strings model is in this case

$$ds^2 = -\left(e^{(at+b)}\right)^2(dx^2 + dy^2 + dz^2) - \left(K_7 e^{(at+b)}\right)^2 dm^2 + dt^2 \quad (21)$$

Here, as the time increases and $a < 0$ the model contracts x,y,z and m directions. The extra dimensions contracts and become unobservable at infinity time. Finally the model reduces to 1-dimensional flat space time.

If $A = (at + b)^n$, Where $a \neq 0, b$ are constant, then $B = K_7 (at + b)^n$

Hence five dimensional strings model is in this case

$$ds^2 = -(at + b)^{2n}(dx^2 + dy^2 + dz^2) - (K_7 (at + b)^n)^2 dm^2 + dt^2 \quad (22)$$

Expansion in the model are x,y,z and m directions.

4. Physical And Geometrical Properties

CASE-1:

1. The spatial volume $V = -g^{\frac{1}{2}} = -A^3 B = \text{Constant}$
The volume of the universe is constant for all t.
2. The scalar expansion $\theta = 0$. Hence the expansion of the universe is zero.
3. The anisotropy σ is defined as

$$\sigma^2 = \frac{1}{2} \left[\left(\frac{g_{00,0}}{g_{00}} - \frac{g_{11,0}}{g_{11}} \right)^2 + \left(\frac{g_{11,0}}{g_{11}} - \frac{g_{22,0}}{g_{22}} \right)^2 + \left(\frac{g_{22,0}}{g_{22}} - \frac{g_{33,0}}{g_{33}} \right)^2 + \left(\frac{g_{33,0}}{g_{33}} - \frac{g_{44,0}}{g_{44}} \right)^2 + \left(\frac{g_{44,0}}{g_{44}} - \frac{g_{00,0}}{g_{00}} \right)^2 \right] = 0$$

$$\sigma^2 = 0$$

$\sigma = 0$ at $t \rightarrow \infty$ $\sigma = 0$ the model is isotropy.

$\frac{\sigma}{\theta}$ is not defined for all t.

CASE-2:

The spatial volume

$$V = -g^{\frac{1}{2}} = -A^3 B = (K_3 t + K_4)^3 (K_5 (K_3 t + K_4)^{-2} + K_6 (K_3 t + K_4))$$

When $t \rightarrow \infty$, $V \rightarrow \infty$ the volume of the universe may be blous up. Again $t \rightarrow 0$ $V = \text{Constant}$ hence the volume of the universe is constant at initial stage.

(i) The scalar expansion

$$\theta = \frac{3\dot{A}}{A} + \frac{\dot{B}}{B} = \frac{3K_3}{K_3t+K_4} + K_3 \left[\frac{K_6 - 2K_5(K_3t+K_4)^{-3}}{K_5(K_3t+K_4)^{-2} + K_6(K_3t+K_4)} \right]$$

(ii) The anisotropy σ is defined as

$$\sigma^2 = \frac{1}{2} \left[\left(\frac{g_{00,0}}{g_{00}} - \frac{g_{11,0}}{g_{11}} \right)^2 + \left(\frac{g_{11,0}}{g_{11}} - \frac{g_{22,0}}{g_{22}} \right)^2 + \left(\frac{g_{22,0}}{g_{22}} - \frac{g_{33,0}}{g_{33}} \right)^2 + \left(\frac{g_{33,0}}{g_{33}} - \frac{g_{44,0}}{g_{44}} \right)^2 + \left(\frac{g_{44,0}}{g_{44}} - \frac{g_{00,0}}{g_{00}} \right)^2 \right] = 0$$

$$\sigma^2 = 0$$

$$\sigma = 0$$

(iii) $\frac{\sigma}{\theta} = 0$

5. Acknowledgement

Authors are very much thankful to the honorable referee for his constructive comments.

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