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# Study of Optical Transfer Function in an Optical System with Gaussian Filter

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**Abstract**

In this paper, we studied the Optical Transfer Function (OTF) in incoherent optical imaging system with Gaussian amplitude apodisation filter. One of the important corollaries of OTF namely, Equivalent Pass-band (EP) is studied. Numerical results are presented different values of spatial frequency and apodised parameter.

**Keywords**: Optical Transfer Function (OTF), Incoherent image, Fourier transform, Equivalent pass-band, spatial frequency, apodisation parameter.

## 1. INTRODUCTION

The Optical transfer function (OTF) in an optical system is a mathematical entity describing how well the subject is transferred into an image via lens. This is a complex-valued function describing the response of an imaging system as a function of spatial frequency. OTF can be resolved into the magnitude and phase components. The magnitude of complex OTF is Modulation Transfer Function (MTF) and phase is known as Phase Transfer Function (PTF). That is, OTF= MTF exp ( iPTF). A perfect optical system will have MTF =1 and PTF =0 for all spatial frequencies. Modulation transfer function describes the response of image of an optical system decomposed into sine waves. Conventionally, the OTF is normalized to unity for zero spatial frequency. In some cases, it is better to consider the un normalized OTF which gives the absolute value of the image signal. In general, the OTF is complex-valued, but it as real-valued in symmetrical optical systems. The OTF is the Fourier transform of the incoherent point spread function. The concept of OTF can be used in Confocal Microscopy and image scanning microscopy. The earliest published work in the core of OTF is due to Mackenzie [1]. In this work, the effect of the recording slit width on high frequency response is studied. The measurement of OTF by auto-correlation of the pupil function was suggested by Hopkins [2]. OTF of diffraction –limited circular apertures have been studied by Barakat and Houston [3]. Srimannarayana [4] et.al.studied the OTF with Straubel filters. Ramanatham [5] et.al, studied imaging characteristics of optical system with Kaiser apodisation filters. Dobryna Zalvindea [6] et.al, analyzed the optical transfer function as image quality parameters of optical elements by employing Wigner distribution function. OTF shaping and depth of focus with phase filters is studied by David Mendlovic and Dina Elkind [7]. L.E.Helseth [8] investigated the OTF of three dimensional display systems. OTF derived from solar adaptive optical system data is studied by Friedrich Woger [9].To the best of our knowledge, no one has analyzed the OTF of symmetrical optical system apodised with Gaussian filter. In this paper, the same is studied.

The rest of the paper is organized as follows. In the section 2, mathematical expression for Optical Transfer Function (OTF) with incoherent light is derived. In the section 3, numerical results are presented, finally conclusion is given in section 4.

## 2. FUNDAMENTALS OF IMAGE FORMATION IN INCOHERENT LIGHT

A schematic representation of an image forming optical system in incoherent illumination is depicted in Fig 1.

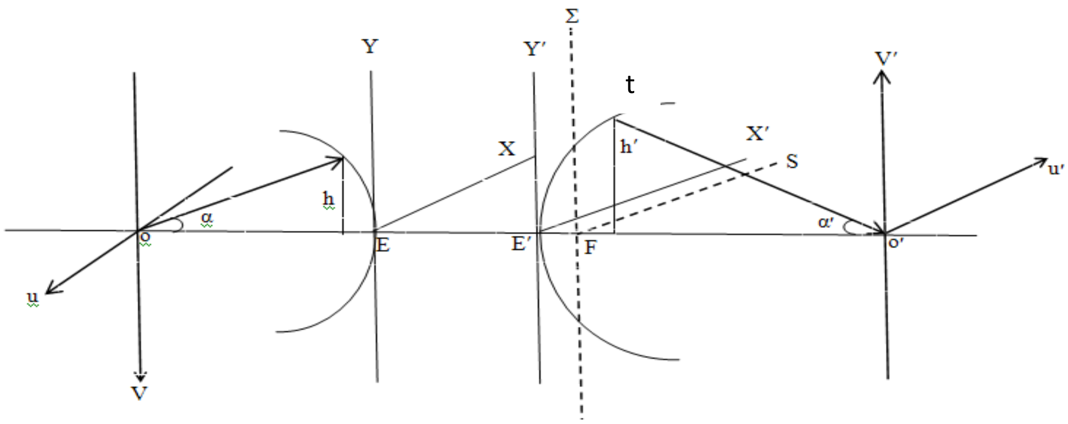


**Fig 1:** Schematic representation of an optical system for imaging an incoherently illuminated object.

The object and image plane are represented by the co-ordinates $(u,v)$and $(u',v')$ respectively with their origins at O and$O'$. The entrance and exit pupils of the optical systems are shown on the axis E and E' .The radius of the reference sphere in the object space is represented as OE. The frequency of the plane $\Sigma$ is shown at the point F on the optical axis and is assumed to be co-incident with exit pupil $E'$ .A ray of light coming from the axial point O in the object plane makes an angle $\alpha$ with the optical axis and it intersects the reference sphere in the object space at a height h. This ray from O converges to $O'$ in the image plane makes an angle $\alpha'$ with the optical axis and it intersects the reference sphere in the object space at a height $h'$. Fractional and reduced co-ordinates of the points and planes of the optical system are defined as follows. The advantage of the transferring the actual co-ordinates into these dimensionless diffraction variables is that it takes care of the problem of magnification produced in the final image. The fractional co-ordinates of points on the entrance and exit pupil of image forming system are defined as

$$X = \frac{a}{R} = \frac{a'}{R'} \qquad Y = \frac{b}{R} = \frac{b'}{R'} \qquad \ldots (1)$$

where (a, b) and$(a', b')$ are the actual Cartesian co-ordinates on the entrance and exit pupils, respectively, R and $R'$ are the respective radii of entrance and exit pupils. The advantage of this transformation is that normal infinite limits of integration in the Fourier transform will be reduced to finite limits. A system with circular symmetry will be considered throughout this work within the aperture limit $x^2 + y^2 = 1$, so that any part of the wavefront which lies outside the circle of unit radius is not transmitted by the optical system. The reduced co-ordinates (u, v) and$(u', v')$ of the points in the object and image planes, respectively are defined as

$$u = \frac{2\Pi}{\lambda}(n \sin \alpha)\zeta \qquad v = \frac{2\Pi}{\lambda}(n \sin \alpha)\eta \qquad \ldots (2)$$

$$\text{and } u' = \frac{2\Pi}{\lambda}(n' \sin \alpha')\zeta' \qquad v' = \frac{2\Pi}{\lambda}(n' \sin \alpha')\eta' \qquad \ldots (3)$$

where $(\zeta, \eta) and (\zeta', \eta')$ are the actual cartesian co-ordinates, $\frac{2\pi}{\lambda}$ and $nsin\alpha$ are propagation constant factor of the illuminating beam and the numerical aperture of the objective of the optical systems respectively. While n and $n'$ are the refractive indices of the object and image space respectively .It is assumed that the wave front associated with a disturbance at O which lies in the object reference sphere has unit amplitude and zero phase. The complex amplitude in the image plane of a point source situated at the origin (0, 0) in the object plane is given by

$$F(u', v') = \iint_{-\infty}^{+\infty} g(x, y) \exp\{-2\pi i\ (u'x, v'y)\}\, dx\, dy\ . \qquad \ldots (4)$$

where g(x,y) function is the pupil function which is finite within the circular aperture and zero outside. Thus the limits $-\infty\ to + \infty$ in the above general integral are restricted by the finite extent of the limiting aperture. The image intensity of a point source situated at the point (0, 0) in the object plane is given by

$$G(u', v') = |F(u', v')|^2 \qquad \ldots (5)$$

In many cases, G (u', v') is not known explicitly. Using the condition of shift invariance, the diffraction image of a point situated at (u, v) in the object plane, is given by $G(u' - u\,, v' - v)$. The convolution of the object intensity distribution and the intensity point spread function gives the resultant intensity $B'(u', v')$ at a point $(u', v')$ in the image plane, thus

$$B'(u', v') = \iint_{-\infty}^{+\infty} B(u, v) G(u' - u, v' - v) du dv \quad , \qquad \ldots (6)$$

where B (u, v) is the object intensity distribution at (u, v) in the object plane. Now, the inverse Fourier transform of the intensity of PSF in the (s, t) is given by

$$h(s, t) = \iint_{-\infty}^{\infty} G(u', v') \exp\{2\pi i(su' + tv')\} du' dv' \quad . \qquad \ldots (7)$$

The co-ordinates (s, t) represent the arbitrary point in the spatial frequency plane. From the equs (4) – (7), we have

$$B'(u', v') = \iint_{-\infty}^{+\infty} h(s\,, t\,) b(s, t) \exp\{2\pi i\, (su' + \; tv')\} ds\, dt \quad . \qquad \ldots (8)$$

The inverse Fourier transform of $B'(u', v')$ is given by

$$b'(s, t) = h(s, t) b(s, t) \qquad \ldots (9)$$

So that

$$h(s, t) = \frac{b'(s, t)}{b(s, t)} \qquad \ldots. (10)$$

h(s, t) is a spatial frequency spectrum of $G'(\,u', v')$ and is defined as the transfer function of the system. Hopkins [2] proposed h(s,t) as a measure of image quality for simple objects. In the case of incoherent light illumination, h(s, t) can be expressed as the auto –correlation integral of the pupil function. Thus

$$h(s, t) = \iint_{-\infty}^{\infty} g(x, y) g(x - s, y - t) dx dy \quad . \qquad \ldots (11)$$

A uniform intensity distribution in the object plane implies a zero frequency $b\,(0\,,0\,)$ in the Fourier spectrum. It is desirable to normalize the optical transfer function to unity at zero frequency. The normalized transmission factor is defined as

$$D(s, t) = \frac{h(s, t)}{h(0,0)} \quad . \qquad \ldots (12)$$

The function D(s,t) is complex valued and has both amplitude and phase components. Thus

$$D(s,t) = T(s,t)\exp(i\theta) \quad , \qquad \text{... (13)}$$

which can be written as

$$D(s,t) = T_N(\omega)\exp[i\theta(\omega)] \quad . \qquad \text{... (14)}$$

In the above, D(s, t) is expressed as a function of the spatial frequency $\omega$, so we have replaced the frequency plane co-ordinates (s, t) with the reduced spatial frequency $\omega$. which can be expressed in terms of the actual line frequency by

$$\omega = \frac{\lambda v}{(NA)} \quad , \qquad \text{... (15)}$$

where $\upsilon$ is the actual line frequency of the object expressed as the number of line per mm or cm, $\lambda$ is the wavelength of the light used ,denominator NA is the numerical aperture of the objective. Following the treatment of Hopkins [2] the one dimensional normalized OTF can be written as

$$T(\omega)_N = \frac{\iint_{-\infty}^{\infty} g(x,y) g[(x-\omega),y] dxdy}{\int\int_{-\infty}^{\infty} |g(x,y)|^2 dxdY} \qquad \text{... (16)}$$

The above expression gives the effect of the auto correlation of pupil function in which the integrand is non zero in the region of overlap of the pupil. In this section we obtain the explicit form of the general auto – correlation integral for the evaluation of the OTF for the specific case of an optical system apodised with Gaussian amplitude filter.

$$\begin{aligned} g(x,y) &= \exp\left(-\frac{r^2}{\sigma^2}\right) \qquad for\ 0 < r < 1 \qquad \text{... (17)} \\ &= 0 \qquad\qquad for\ r > 1\ , \end{aligned}$$

where $\sigma$ is the apodisation parameter which controls transmission of the pupil. From equations (16) and (17), we have

$$T(\omega)_N = \frac{\iint_{-\infty}^{\infty} exp\left(-\frac{(x^2+y^2)}{\sigma^2}\right) exp\left(-\frac{\{(x-\omega)^2+y^2\}}{\sigma^2}\right) dxdy}{\int\int_{-\infty}^{\infty} \left|exp\left(-\frac{(x^2+y^2)}{\sigma^2}\right)\right|^2 dxdY} \quad , \quad \text{... (18)}$$

where, the limits of integration decided by the fact that the pupil transmission is zero for value of $x^2 + y^2 > 1$. Therefore the infinite limits of integral can be replaced by finite limits. Spatial frequency for which the two pupils completely overlap with each other ($\omega = 0$), partially overlap with each other ($\omega < 2$) and non overlapping case ($\omega = 2$). These cases are depicted in Fig 2:

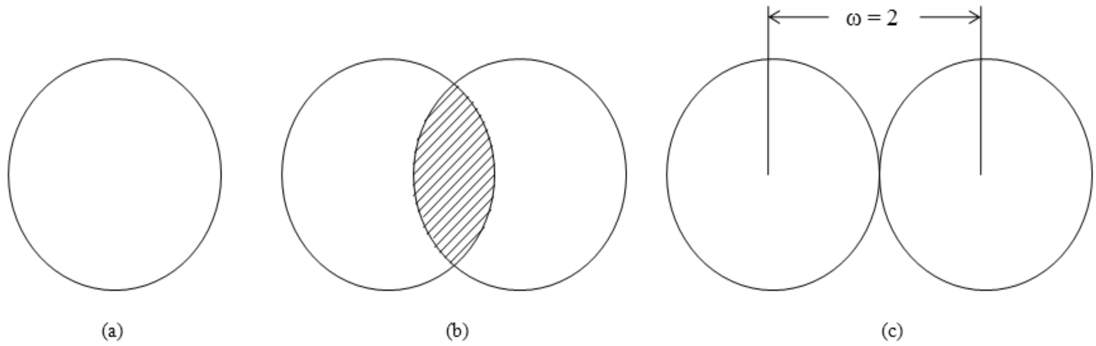


**Fig 2:** Possible cases of spatial frequency

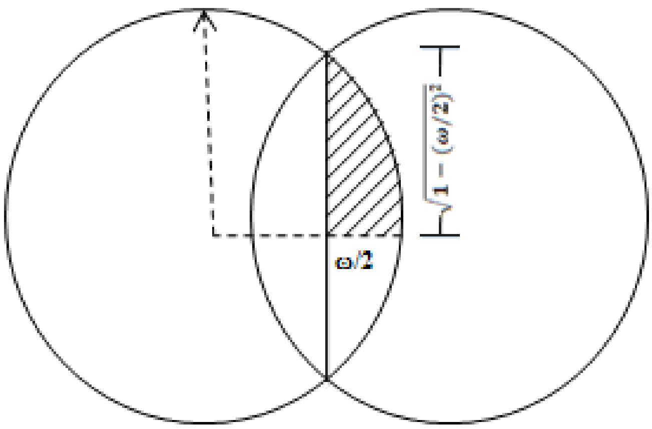


**Fig 3:** Auto correlation of the pupil function. Shaded region is the domain of integral of normalized OTF.

Because of the symmetry of domain of integration as depicted in the Fig 3, the equation (18) reduces to

$$T(\omega)_N = \frac{\int_{\frac{\omega}{2}}^{1} dx \int_0^{\sqrt{1-x^2}} \exp\left\{-\frac{(x^2+y^2)}{\sigma^2}\right\} \exp\left\{-\frac{((x-\omega)^2+y^2)}{\sigma^2}\right\} dy}{\int_0^1 dx \int_0^{\sqrt{1-x^2}} \left|\exp\{-\frac{(x^2+y^2)}{\sigma^2}\right|^2 dy} \quad \ldots (19)$$

The limits in the numerator are due to fact that the integration need to be carried out only for one quarter part of the entire area of overlap as shown in Fig 3. The denominator in the above expression is a normalizing factor required to make the OTF value equal to unity at zero spatial frequency. Equivalent Pass-band (EP) also called the 'Structural Resolution' [10] is an important parameter for the evaluation of the performance of an Optical system. This is useful criterion for the purpose of determining how closely an apodised system approaches to unapodised system whose modulation transfer function is controlled by the effects of diffraction. By using numerical methods, Levi [11] calculated the EP of an aberration free lens. The EP of a lens is defined as the integral of the square of the Modulation Transfer Function (MTF) of the lens [12]. Thus, the EP can be expressed as

$$EP = \frac{1}{2}\int_0^2 [T(\omega)_N]^2 d\omega \qquad \ldots (20)$$

## 3. RESULTS AND DISCUSSION

In this section, the results of incoherent OTF for the rotationally symmetric Gaussian filters are discussed. The transfer function has been computed over the range of spatial frequencies from 0 to 2.0 for apodisation parameter $\sigma$ from 0.1 to 1.0 in step of 0.1, and 3.08607. The numerical results of $T(\omega)_N$ are presented in Fig 4. From the curves, it is observed that there is an enhancement of response to lower frequencies for all values $\sigma$, meaning that OTF of the apodised system is larger than that of Airy system. The cut off frequencies show a gradual increase with increasing values of $\sigma$ from 0.1 to 1.0. In the case of $\sigma$ =3.08607, cut off frequency approaches to 2.0. In the case of $\sigma$ =0 (Airy case) as $\omega = 0$, the value of OTF is 1. In the case of $\sigma$ =0 and other values of $\omega$, OTF becomes zero. The little enhancement of OTF in low frequency stands in poor comparison to the large suppression in high frequency region. The EP has been calculated for each value of $\sigma$ varying from 0 to 1 in step of 0.1 results are presented Fig 5. EP value is 0.2978 for $\sigma$ =0.9 and for higher values of $\sigma$ EP is decreasing. This is optimum value of apodisation at which the transmission of signal power is maximum.

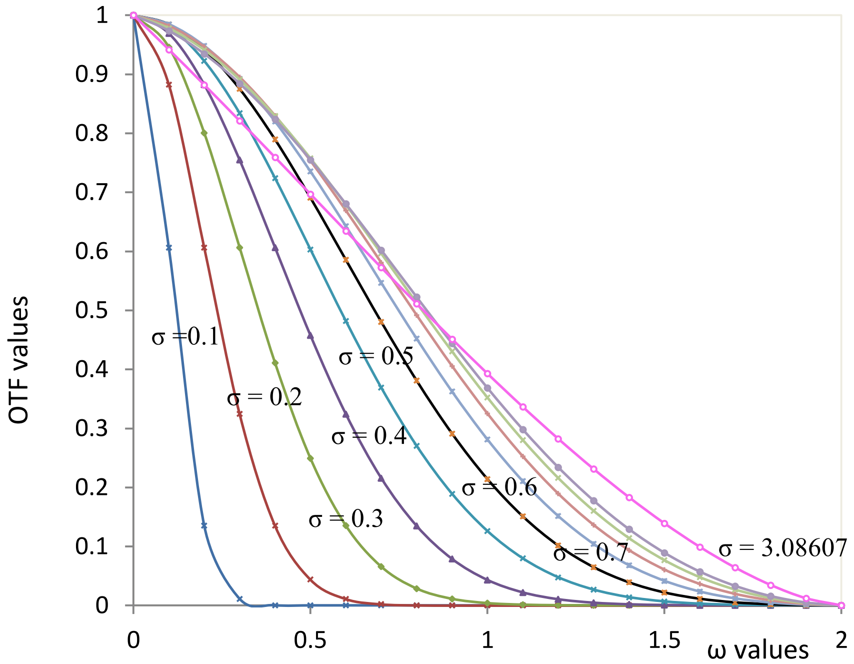


**Fig 4:** Optical Transfer Function of Gaussian apodizers

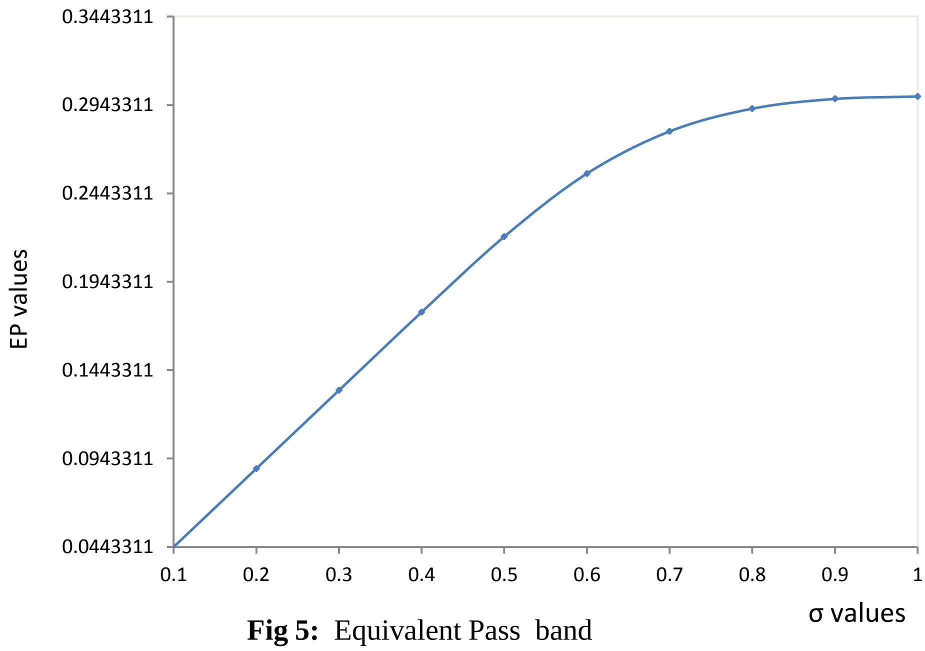


**Fig 5:** Equivalent Pass band

## 4. CONCLUSION

The complex OTF and EP of symmetrical optical system with incoherent light of illumination is studied using Gaussian amplitude filter. These parameters are computed against the spatial frequency for different values of apodisation parameter. This kind of analysis is useful in confocal microscopy, photography, and signal processing system.

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