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A Dynamical Model of the Miniature Solar System

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Abstract

In the University of Colorado Miniature Solar System Model (Model A), the distances are scaled down by a factor of 10^{-10} but time remains unaltered. Model A is virtually a stationary one, in which the planets revolve around the Sun in their actual revolution times. In the quest for a more dynamic model, one is apt to try a model (Model C), in which both distance and time are reduced by the same factor of 10^{-10}. But in Model C, the revolution periods of the inner planets are of the order of milliseconds, while Pluto takes just under a second to complete one revolution. In other words, Model C is an ultrafast one which is unfeasible for demonstration purposes apart from the constructional difficulties it would present. A compromise between the two extreme models is Model B, where time is reduced by a factor of 10^{-5}. In Model B, Mercury revolves once around the Sun in 76s, Pluto revolves once in about 21.7h, while Earth rotates once in under a second. Model B is the most appropriate dynamical model amongst the three. The scale factors with which the common dynamical quantities must be multiplies under the various models are determined. In all three models, the fourth dimension in Minkowski space is reduced by the same factor of 10^{-10}.

1. THE MINIATURE SOLAR SYSTEM MODEL

Scaled-down models of large structures are often made for perspective and demonstrational purposes. The first ***scale-model of the solar system*** was constructed at the University of Colorado in 1987 for educational purposes and perception of the vastness of space in which we live [1]. In that model, the actual ***distances are scaled down by a factor of 10^{-10}*** or 1 in 10 billion [1]. In that model, the Sun is the size of a grapefruit (diameter 14 cm); the Earth (diameter 1 mm) is located 15 m from the Sun; and the nearest star system (Alpha Centauri) is some 4,000 km away from the Sun [1]. Similar solar system models (called ***Voyage Scale Model Solar System***) were open in the National Mall in Washington, DC [2], Kansas City [3], Houston [4] and Corpus Christi [5]. Figure 1 (from [2]) captures the model in the National Mall, Washington.

Fig. 1. Voyage Scale Model Solar System in Washington, DC, U.S.A.

For practical reasons, the above models are stationary ones. The planets are arranged on a straight line from the Sun, whose sizes and distances from the Sun are apparent, but whose angular motions (viz., revolution and rotation) are suppressed. This paper explores the possibility of constructing a ***dynamical model of the miniature solar system*** in which the angular motions of the Sun and the planets can be demonstrated.

2. PROPOSED DYNAMICAL MODELS OF THE MINIATURE SOLAR SYSTEM

At the outset, it must be stated that for constructing a dynamical model of the miniature solar system, a canvass of the size of a football field will be needed. Next, the proceedings will be greatly facilitated if the orbits of the planets are assumed to be coplanar. This assumption is fairly reasonable with the exception of Pluto (now demoted to the status of a dwarf planet), whose orbit is significantly inclined from the ecliptic. Finally, a proper ***scaling factor for the time*** must be assumed or determined if possible. In this paper, three dynamical models of the miniature solar system are proposed depending upon the scaling factor of time: (1) **Model A** (University of

Colorado Model), in which time remains unaltered, scaling factor 1; (2) **Model C**, in which time is scaled down by a factor of 10^{-10}; and (3) **Model B**, in which time is scaled down by a factor of 10^{-5}. We determine how the geometrical and dynamical quantities are scaled (either up or down or unchanged) in each model from definitions or physical laws. For this purpose, a dimensional analysis is found to be useful [6], by which the physical quantities are expressed in terms of the ***fundamental units*** of ***length*** (L), ***time*** (T) and ***mass*** (M). Next, a few examples of the dynamical variables are determined to help select the most suitable model.

3. MODEL A

The first model is the proposed University of Colorado model [1] with the time element incorporated in it. In that model, the speed of light is scaled down by the same factor of 10^{-10} as are the distances [1]. This implies that the ***time must remain unaltered***. Denoting the original quantities as unprimed and the new quantities in the miniature coordinate system as primed, we have for ***time***: $t' = t$. For the ***speed of light***, $c' = 10^{-10} c$. The ***fourth coordinate in Minkowski space*** is defined as $\tau = ict$, where $i = \sqrt{-1}$, whence $\tau' = 10^{-10}\,\tau$. For geometrical quantities of ***area*** and ***volume***, we have obviously: $A' = 10^{-20} A$; and $V' = 10^{-30} V$. For the ***areal velocity*** (area swept by a planet at the Sun) of Kepler's second law, we have: $dA/dt' = 10^{-20}\, dA/dt$. Since the ***orbital angular momentum*** of a planet is proportional to the areal velocity, one gets: $\ell' = 10^{-20}\,\ell$.

In order to determine the scale factor for ***mass***, it is prudent to keep the ***density*** unaltered. To digress, there are two main classes of large objects in the solar system: (1) The ***Sun*** and the ***Jovian planets***, whose densities are around 1 g/cc; and (2) The ***Terrestrial planets*** whose densities range between 3.9 g/cc (for Mars) to 5.5 g/cc (for the Earth), depending upon the iron/nickel content in their interiors. Even though the Jovian planets are called 'gas planets', their interiors are believed to be liquid and/or solid, which are ***incompressible matter***, just as the interiors of the 'rocky' Terrestrial planets are. Hence it will only be rational to keep the density the same, which will mean that the ***mass will vary as the volume*** of the celestial objects in the solar system: $m' = 10^{-30} m$. This will be true for Models B and C also.

Having assumed the scale factor for time and determined the scale factor for mass, one can now proceed to find the scale factors of any and all dynamical quantity from ***definition*** or a ***law of nature***. As examples, the scale factor for ***momentum*** can be determined from definition, whereas the scale factor for ***force*** can be determined from ***Newton's second law of motion***:

$$f = ma\,, \tag{1}$$

where a is the ***acceleration*** produced by the force. Applying dimensional analysis, we get: $f' = 10^{-40} f$. As a particular example, the ***centripetal force*** can be determined from

Huygen's formula:

$$f = \frac{mv^2}{r} \ , \tag{2}$$

where r is the radius of the circular path. Equation (2) yields the same scaling factor: $f' = 10^{-40} f$. Likewise, the ***gravitational force*** between two masses M and m, separated by a distance r is given by ***Newton's law of gravitation***:

$$f = \frac{GMm}{r^2} \ . \tag{3}$$

Since, this is alsoexpected to yield the same scaling relation for the force, we can use Eq. (3) to determine the scale factor for the ***universal gravitational constant***: $G' = G$.

The rest of the relevant dynamical quantities are determined in this fashion and the results are entered in Table I. For example, the ***escape velocity from the Earth*** is determined from the equation:

$$v = \sqrt{\frac{2GM_0}{r_0}} \ . \tag{4}$$

Here M_0 is the mass of the Earth and r_0 is its radius.

Table I indicates that in Model A, all velocities are slowed down by a factor of 10^{-10}, which is a tremendous reduction. The average orbital speed of a planet can readily be calculated from the planet's time period and average distance from the Sun. In Model A, the average speed of the fastest planet Mercury is only 4.8×10^{-4}cm/s; and that of the Earth is only 3×10^{-4}cm/s [7]. The speed of light (which is the fastest permissible speed in the universe) in Model A is merely 3cm/s [1]. Thus, in Model A, the planets move imperceptibly, and even with the time element added to it, Model A is virtually a stationary one.

4. MODEL C

In search of a more dynamical miniature solar system model, we first consider Model C, in which both the distances and time are altered by the same factor of 10^{-10}. The scale factors of the dynamical variables are calculated accordingly and entered in Table I. In Model C, the orbital speeds of the planets remain the same. However, their revolution and rotation periods are reduced by a factor of 10^{-10}. In such a model, the revolution period of Mercury will be less than a millisecond; that of the Earth merely 3 milliseconds; while Pluto takes just under a second to complete one revolution around the Sun. In other words, Model C would be an ultrafast one which will be unfeasible for demonstration purposes, apart from the constructional difficulties it would present. It would be a model in the diametrically opposite end of Model A.

Table I. Scaling Factor of Physical Quantities

Physical Quantity	Dimension	**Model A**	**Model B**	**Model C**
Length, Distance	L	10^{-10}	10^{-10}	10^{-10}
Time	**T**	**1**	$\mathbf{10^{-5}}$	$\mathbf{10^{-10}}$
Fourth Dimension	L	10^{-10}	10^{-10}	10^{-10}
Velocity, Speed	LT^{-1}	10^{-10}	10^{-5}	1
Acceleration	LT^{-2}	10^{-10}	1	10^{10}
Area	L^2	10^{-20}	10^{-20}	10^{-20}
Volume	L^3	10^{-30}	10^{-30}	10^{-30}
Areal Velocity	L^2T^{-1}	10^{-20}	10^{-15}	10^{-10}
Density	ML^{-3}	1	1	1
Mass	M	10^{-30}	10^{-30}	10^{-30}
Momentum	MLT^{-1}	10^{-40}	10^{-35}	10^{-30}
Force	MLT^{-2}	10^{-40}	10^{-30}	10^{-20}
Centripetal Force	MLT^{-2}	10^{-40}	10^{-30}	10^{-20}
Gravitational force	MLT^{-2}	10^{-40}	10^{-30}	10^{-20}
Gravitational Constant	$M^{-1}L^3T^{-2}$	1	10^{10}	10^{20}
Angle	0	1	1	1
Angular Velocity	T^{-1}	1	10^{5}	10^{10}
Angular Acceleration	T^{-2}	1	10^{10}	10^{20}
Moment of Inertia	ML^2	10^{-50}	10^{-50}	10^{-50}
Angular Momentum	ML^2T^{-1}	10^{-20}	10^{-15}	10^{-10}
Kinetic Energy	ML^2T^{-2}	10^{-50}	10^{-40}	10^{-30}
Time Period	T	1	10^{-5}	10^{-10}
Revolution Period	T	1	10^{-5}	10^{-10}
Rotation Period	T	1	10^{-5}	10^{-10}
Orbital Speed of Mercury		$4.8\text{x}10^{-4}$cm/s	48cm/s	48km/s
Orbital Speed of Earth		$3\text{x}10^{-4}$cm/s	30cm/s	30km/s
Orbital Speed of Pluto		$4.7\text{x}10^{-5}$cm/s	4.7cm/s	4.7km/s
Revolution Period of Mercury		88d	76s	$7.6\text{x}10^{-4}$s
Revolution Period of Earth		365d	5m 15.36s	$3.15\text{x}10^{-3}$s
Revolution Period of Pluto		248y	21h 43m 29s	.782s
Rotation Period of Earth		1d	.864s	$8.64\text{x}10^{-6}$s
Speed of light		3cm/s	3km/s	$3\text{x}10^{5}$km/s
Velocity of Escape from Earth		$1.1\text{x}10^{-4}$cm/s	11cm/s	$1.1\text{x}10^{4}$m/s

5. MODEL B

A compromise between the two extreme models (A and C) is Model B, where distances are reduced by the same factor of 10^{-10} while time is reduced by a factor of 10^{-5}. Consequently, both time and velocities are reduced by the same factor of 10^{-5}. The orbital speeds of the earth is 30cm/s; the rotation period of the Earth is just under a second; while the revolution period of the Earth is just over 5m. In other words, Model B is neither ultrafast (like Model C), nor excruciatingly slow (like Model A). This is the dynamical model in which the dynamical variables are most reasonable. It is also the model which is amenable to actual construction.

6. REMARKS

It is interesting to note that in all three models, the fourth dimension τ of Minkowski space is the same, i.e., the line element $ds^2 = dr^2 + d\tau^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$ is invariant. In that respect, all three models pass the special relativistic test. From a practical point of view, however, Model B is the most appealing of the three and is selected as our dynamical model of the miniature solar system.

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