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Optimizing Advertising, Pricing and Inventory Policies in VMI Production Supply Chains with Compensating Cost in Fuzzy Environment

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Abstract

In this paper, we developed New fuzzy VMI (Vendor Managed Inventory) supply chain problem with one manufacturer and multiple retailers as a Stackelberg game model where the manufacturer leads and all retailers follow to determine their own optimal product marketing (advertising and pricing) and inventory policies. A computational algorithm has been proposed to solve this game model based on the theoretical analysis of the best response functions with a generic demand function. The model analysis is specialized in a case with the Cobb–Douglas demand function. Here we find out the fuzzy total inventory cost for the finished product and raw materials, fuzzy Net profit for manufacturer and all retailers. Demand rate, Advertising, Holding and order cost, retail and wholesale price of supplier and retailers, fraction of backlogging rate, common replenishment cycle time are taken as triangular fuzzy numbers. Graded mean integration representation method is used for defuzzification.

Keywords: Vendor managed inventory, Supply chain, Triangular fuzzy number

Mathematics Subject Classification: 03E72, 90B05

1. Introduction

The fundamental purpose of supply chain management is to efficiently coordinate material, information, and financial flows so as to reduce risks of demand-supply mismatches. These risks can be mitigated through implementing optimal or nearoptimal inventory policies, coordinating supply chain members, sharing demand and projected order information, and exercising operational hedging strategies. Hoque [03] presented a single-vendor multi customer system and considered vendor' setup and inventory holding costs.

Evidence has shown that VMI can improve supply chain performance by decreasing inventory levels and increasing fill rates; as a result, industry use of VMI has grown over time. VMI is a collaborative commerce initiative where suppliers are authorized to manage the buyer's inventory of stock-keeping units. It integrates operations between suppliers and buyers through information sharing and business process reengineering. By using information technologies, such as Electronic Data Interchange (EDI) or Internet-based XML protocols, buyers can share sales and inventory information with suppliers on a real time basis. Suppliers can then use this information to plan production runs, schedule deliveries, and manage order volumes and inventory levels at the buyer's stock-keeping facilities. The potential benefits from VMI are very compelling and can be summarized as reduced inventory costs for the supplier and buyer and improved customer service levels, such as reduced order cycle times and higher fill rates.

Some researchers [07] consider vendors as manufacturers by involving the procurement of one kind of raw material in VMI systems. However, they often omit two important issues. One is that supply chain members, often as separate and independent economic entities, remain autonomous on advertising and pricing policies. The other is that the vendor, often a manufacturer, needs to procure multiple raw materials to make products.

In this paper we developed how a manufacturer and its retailers interact with each other in order to optimize their individual fuzzy net profits by adjusting product marketing and inventory policies in an information-asymmetric VMI supply chain. The manufacturer purchases multiple components or raw materials according to the BOM (bill of materials) of the finished product, produces the finished product and distributes it to its retailers. This supply chain has three levels of retailers, the manufacturer, and the suppliers of raw materials [04]. The manufacturer produces and supplies a single product at the same fuzzy wholesale price to multiple retailers who then sell the product in dispersed and independent markets at fuzzy retail prices.

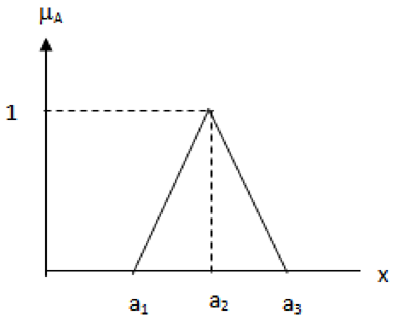
The manufacturer determines its fuzzy wholesale price, its fuzzy advertising investment, replenishment cycles for the raw materials and finished product, and backorder quantity to maximize its profit. Retailers in turn consider the replenishment policies and the manufacturer's promotion policies and determine the optimal fuzzy retail prices and fuzzy advertisement investments to maximize their profits. This problem is modeled as a Stackelberg game where the manufacturer is the leader and retailers are followers. An algorithm has been proposed to search the Stackelberg equilibrium.

2. Methodology

2.1. Triangular Fuzzy Number

The fuzzy set $\breve{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, \text{otherwise} \end{cases}$$



2.2. The Function Principle

The function principle was introduced by Chen [1] to treat fuzzy arithmetical operations. This principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A}$ = (a_1, a_2, a_3,) and $\tilde{B}$ = (b_1, b_2, b_3) are two triangular fuzzy numbers. Then

The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}$ + $\tilde{B}$ = (a_1+b_1, a_2+b_2, a_3+b_3) where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}$ x $\tilde{B}$ = (c_1, c_2, c_3) where T = { a_1b_1, a_1b_3, a_3b_1, a_3b_3} c_1 = min T, c_2 = a_2b_2, c_3 = Max T

If a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then $\tilde{A}$ x $\tilde{B}$ = (a_1b_1, a_2b_2, a_3b_3)

$\tilde{B}$ = ($-b_3$, $-b_2$, $-b_1$) then the subtraction of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}$ - $\tilde{B}$ = (a_1-b_3, a_1-b_2, a_3-b_1) where a_1,a_2,a_3, b_1, b_2, b_3 are any real numbers

The division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}/\tilde{B} = \left(a_1/b_3, a_2/b_2, a_3/b_1\right)$

For any real number K, K$\tilde{A}$ = (Ka_1, Ka_2, Ka_3) if K > 0

K$\tilde{A}$ = (Ka_3, Ka_2, Ka_1) if K < 0

2.3. Graded Mean Integration Representation Method

If $\tilde{A}$ = (a_1, a_2, a_3) is a triangular fuzzy number then the graded mean integration representation of $\tilde{A}$ is given by $P(\tilde{A}) = a_1 + 4a_2 + a_3/6$

2.4. Cobb-Douglas function

The product demand of retailer j, $\tilde{D}_j$ for j=1,2,…n is dependent on its retail price ($\tilde{P}_{rj}$) , the manufacturer's Advertising cost ($\tilde{A}$), and the retailer i's Advertising cost ($\tilde{a}_j$) .

$\widetilde{D}_j\left(\tilde{P}_{rj}, \tilde{a}_j, \tilde{A}\right) = K_j \frac{\tilde{a}_j^{\alpha_j} \tilde{A}^{\beta_j}}{\widetilde{Pr}_j^{\rho_j}}$, j = 1,2 … n. Where K_j is positive constant representing the market scale of retailer j; α_j, β_j and ρ_j represent the elasticity parameters of $\tilde{a}_j, \tilde{A}$ and $\tilde{P}_{rj}$ [02,09].

2.5. The Stackelberg Game Model

This section models a non-cooperative stackelberg game where the manufacturer acts as the leader and retailers act as the followers. Their net profits are considered as the players' payoff/utility functions for maximization. The manufacturer's decisions and retailer's decisions are determined. In order to make the stackelberg game model more easier to follow and more applicable, this model will be started with a generic demand function D_j. The model and their results will be instantiated with a Cobb-Douglas demand function as an application of our proposed model in the next section. We assume that the inventory levels for the retailers and the manufacturer have the trend shown in Fig.1. For each retailer, replenishment rate is infinite and backorder shortage is allowable but manufacturer's replenishment rate is finite and shortage is unallowable [02,05,09].

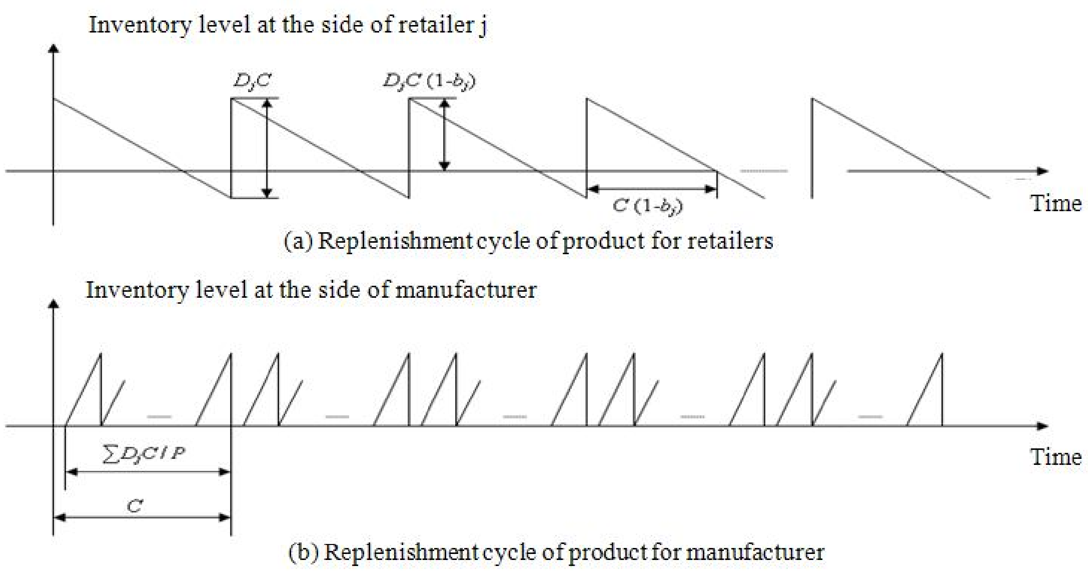


Fig. 1: The product inventory level

2.6. The Stackelberg Equilibrium

The stackelberg equilibrium is obtained using a backward procedure. Based on this procedure, the followers (retailer) problem must be solved first to get the response functions of the leader's (manufacturer) decisions. In the next step, the manufacturer's decision problem is solved by attending all possible reactions of the followers to maximize the net profit. Every follower's optimal response can be determined by considering the manufacturer's decisions as its input parameters. Finally, the leader finds its optimal decision by assuming that the followers take the optimal response.

2.7. Assumptions

In this supply chain consider one manufacturer (vendor or supplier) and multiple retailers who are involved in producing, delivering and selling only one type of finished product.

Retailer's markets are assumed to be geographically dispersed and independent of each other.

The fuzzy demand rate in each local retail market is assumed to be an increasing function of the fuzzy Advertising costs made by the corresponding local retailer and the manufacturer and a decreasing function of the fuzzy retail price.

A common replenishment cycle policy is adopted by the supplier to manage the inventories of the product. Each retailer pays the fuzzy inventory cost based on the fuzzy demand rate to the supplier.

Based on the VMI strategy, the supplier is responsible for the chain wide two-echelon inventories which include the finished product's inventories at the retailer's sides and the supplier's side.

2.8. Notations

Indices

m - number of retailers

i = 1,2,. . . ,m , index of the retailers or markets

l - number of raw materials

j = 1,2, . . . , l , index of raw materials

Decision variables of retailer i and manufacturer

a_i - Advertising cost for retailer i (Rs./time)

P_{ri} - retail price charged by retailer i (Rs./unit)

A - Advertising cost for manufacturer (Rs./time)

b_i - fraction of backlogging rate in a cycle for retailer i (Rs./time)

C - Common replenishment cycle time for the finished product

P_m - wholesale price of the finished product set by the manufacturer (Rs./unit)

n_j - cycle factor n_j for raw material j which is an integer.

n_jC - order / procurement cycle time for raw material j

Parameters

C_m - manufacturing cost for per unit finished product (Rs./unit)

C_{rj} - price for per unit raw material j (Rs./unit)

H_{bi} - holding cost paid by the manufacturer at retailer i's side (Rs./unit/time)

H_m - holding cost per unit finished product of inventory at the manufacturer's side (Rs./unit/time)

H_{rj} - holding cost per unit raw material j at the manufacturer (Rs./unit/time)

K_i - a positive constant representing the market scale of retailer i in the Cobb–Douglas function

L_{ri} - backorder cost paid by the manufacturer to retailer i's side (Rs./unit/time)

M_j - the usage factor of raw material j representing the quantity of raw material j required for producing each finished product

P - production rate of the finished product for the manufacturer, which is a known constant and $\sum_{i=1}^{m} D_i \leq P$

S_{bi} - fixed order cost paid by the manufacturer for retailer i's side (Rs. for one time)

S_m - fixed order cost for a common cycle time for the finished product at the manufacturer's side (Rs. /order setup)

S_{rj} - fixed order cost for the procurement of raw material j (Rs./order setup)

φ_i - transportation cost per unit finished product shipped from the manufacturer to retailer i (Rs./unit)

γ_i - inventory cost for retailer i (Rs./unit/time)

α_i - retailer i's advertising (a_i) elasticity of demand in the Cobb–Douglas function

β_i - manufacturer's advertising (A) elasticity of the demand in the Cobb–Douglas function

ρ_i - the price elasticity of retailer i's demand in the Cobb–Douglas function

Functions

CC_i - compensating cost for the manufacturer to retailer i (Rs. /time)

$D_i = D_i(P_{ri}, a_i, A)$ - demand rate of the finished product in market i served by retailer i, a decreasing and convex function of P_{ri} and an increasing and concave function of a_i and A (unit/time)

HIC_{rj} - total holding inventory cost for raw material j (Rs. /cycle)

TDC_p - total direct cost for the finished product (Rs. /time)

TIC_r - total inventory cost for all raw materials (Rs. /time)

TIC_p - total inventory cost for the finished product (Rs. /time)

TIC - total inventory cost for the finished product and raw materials (Rs. /time)

$TIDC_p$ - total indirect cost for the finished product (Rs. /time)

NP_{ri} - net profit for retailer i (Rs. /time)

NP_m - net profit for the manufacturer (Rs. /time)

Fuzzy Decision variables of retailer i and manufacturer

$\tilde{\alpha}_i$ - fuzzy Advertising cost for retailer i

$\tilde{P}_{ri}$ - fuzzy retail price charged by retailer i

$\tilde{A}$ - fuzzy Advertising cost for manufacturer

$\tilde{P}_m$ - fuzzy wholesale price of the finished product set by the manufacturer

$\tilde{b}_i$ - fraction of fuzzy backlogging rate in a cycle for retailer i

$\tilde{C}$ - fuzzy common replenishment cycle time for the finished product

Fuzzy Parameters

$\tilde{C}_m$ - fuzzy manufacturing cost for per unit finished product

$\tilde{C}_{rj}$ - fuzzy price for per unit raw material j (Rs./unit)

$\tilde{H}_{bi}$ - fuzzy holding cost paid by the manufacturer at retailer i's side

$\tilde{H}_m$ - fuzzy holding cost per unit finished product of inventory at the manufacturer's side

$\tilde{H}_{rj}$ - fuzzy holding cost per unit raw material j at the manufacturer

$\tilde{L}_{ri}$ - fuzzy backorder cost paid by the manufacturer to retailer i's side

$\tilde{M}_j$ - fuzzy usage factor of raw material j representing the quantity of raw material j required for producing each finished product

$\tilde{P}$ - fuzzy production rate of the finished product for the manufacturer

$\tilde{S}_{bi}$ - fuzzy fixed order cost paid by the manufacturer for retailer i's side

$\tilde{S}_m$ - fuzzy fixed order cost for a common cycle time for the finished product at the manufacturer's side

$\tilde{S}_{rj}$ - fuzzy fixed order cost for the procurement of raw material j

$\tilde{\varphi}_i$ - fuzzy transportation cost per from the manufacturer to retailer i

$\tilde{\gamma}_i$ - fuzzy inventory cost for retailer i

Fuzzy functions

$\widetilde{CC}_i$ - fuzzy compensating cost for the manufacturer to retailer i

$\widetilde{D}_i$ - fuzzy demand rate of the finished product in market i served by retailer i

$\widetilde{HIC}_{rj}$ - fuzzy total holding inventory cost for raw material j

$\widetilde{TDC}_p$ - fuzzy total direct cost for the finished product

$\widetilde{TIC}_r$ - fuzzy total inventory cost for all raw materials

$\widetilde{TIC}_p$ - fuzzy total inventory cost for the finished product

$\widetilde{TIC}$ - fuzzy total inventory cost for the finished product and raw materials

$\widetilde{TIDC}_p$- fuzzy total indirect cost for the finished product

$\widetilde{NP}_{ri}$ - fuzzy net profit for retailer i

$\widetilde{NP}_m$ - fuzzy net profit for the manufacturer

3. Fuzzy Mathematical Model

3.1. Fuzzy Net Profit of Each Retailer

The payoff function (net profit) for each player is equal to its revenue minus its total cost. Fuzzy revenue of retailer i is $\tilde{P}_{ri}\widetilde{D}_i$. Fuzzy Product cost is $\tilde{P}_m\ \widetilde{D}_i$. Fuzzy Advertising cost is $\tilde{a}_i$. Fuzzy Inventory cost is $\gamma_i\ \widetilde{D}_i$.

$$\widetilde{NP}_{ri} = \tilde{P}_{ri}\widetilde{D}_i - \tilde{P}_m\widetilde{D}_i - \gamma_i\widetilde{D}_i - \tilde{a}_i = \left(\tilde{P}_{ri} - \tilde{P}_m - \gamma_i\right)\widetilde{D}_i - \tilde{a}_i \quad \dots (1)$$

3.2. Fuzzy Net Profit of the Manufacturer

The total revenue for the manufacturer comes from the selling of the finished product to its retailers at wholesale price $\tilde{P}_m\ (= \sum_{i=1}^{m} \tilde{P}_m\widetilde{D}_i)$. The fuzzy total cost is divided into the fuzzy total direct cost ($\widetilde{TDC}_m$) and fuzzy total indirect cost ($\widetilde{TIDC}_m$). The fuzzy total direct cost per unit time consists of fuzzy production cost, fuzzy transport cost and raw materials fuzzy procurement cost.

$$\widetilde{TDC}_p = \sum_{i=1}^{m} \widetilde{D}_i \left(\tilde{C}_m + \phi_i + \sum_{j=1}^{l} \widetilde{M}_j\tilde{C}_{rj} \right) \quad \dots\dots (2)$$

The total indirect cost for the manufacturer is includes fuzzy advertising expenditure ($\tilde{A}$), fuzzy compensation costs ($\widetilde{CC}_i$) to retailers under VMI, and fuzzy total inventory cost ($\widetilde{TIC}$).

$$\widetilde{TIDC}_p = \sum_{i=1}^{m} \widetilde{CC}_i + \widetilde{TIC} + \tilde{A} \quad \dots\dots (3)$$

Compensation cost from the manufacturer to retailer i,

$$\widetilde{CC}_i = \frac{\tilde{S}_{bi}}{\tilde{C}} + \frac{\widetilde{D}_i(1-\tilde{b}_i)^2\tilde{C}}{2}\widetilde{H}_{bi} + \frac{\widetilde{D}_i\tilde{b}_i^2\tilde{C}}{2}\tilde{L}_{ri} - \gamma_i\widetilde{D}_i \quad \ldots\ldots (4)$$

The manufacturer manages the ordering and inventory levels and backorder cost for each retailer and therefore must pay the Compensation cost.

$$\widetilde{TIC}_p = \frac{1}{\tilde{C}}\left[\tilde{S}_m + \widetilde{H}_m\sum_{i=1}^{m}\frac{\widetilde{D}_i^{\,2}\tilde{C}^2}{2\tilde{P}}\right] \quad \ldots\ldots (5)$$

The manufacturer's holding cost ($\widetilde{HIC}_{rj}$) for raw material j in replenishment cycle $n_j\tilde{C}$ is,

$$\widetilde{HIC}_{rj} = \widetilde{H}_{rj}\left[\frac{n_j}{2}\left(\frac{\sum_{i=1}^{m}\widetilde{D}_i\tilde{C}}{\tilde{P}}\right)\left(\widetilde{M}_j\sum_{i=1}^{m}\widetilde{D}_i\tilde{C}\right) + \sum_{k=1}^{n_j-1}\left(k\tilde{C}\widetilde{M}_j\sum_{i=1}^{m}\widetilde{D}_i\tilde{C}\right)\right]$$

$$= \frac{n_j\widetilde{M}_j\widetilde{H}_{rj}}{2\tilde{P}}\left(\sum_{i=1}^{m}\widetilde{D}_i\tilde{C}\right)^2 + \frac{n_j(n_j-1)\widetilde{M}_j\widetilde{H}_{rj}}{2}\sum_{i=1}^{m}\widetilde{D}_i\tilde{C}^2 \quad \ldots\ldots (6)$$

Fuzzy total inventory cost for all raw materials per unit time is,

$$\widetilde{TIC}_r = \sum_{j=1}^{l}\frac{1}{n_j\tilde{C}}\left(\tilde{S}_{rj} + \widetilde{HIC}_{rj}\right)$$

$$= \frac{1}{\tilde{C}}\sum_{j=1}^{l}\frac{\tilde{S}_{rj}}{n_j} + \frac{\tilde{C}}{2}\sum_{j=1}^{l}\left[\widetilde{M}_j\widetilde{H}_{rj}\sum_{i=1}^{m}\widetilde{D}_i\left(n_j - 1 + \frac{\sum_{i=1}^{m}\widetilde{D}_i}{\tilde{P}}\right)\right] \quad \ldots\ldots (7)$$

Fuzzy total inventory cost for all materials and the finished product at the manufacturer's side is

$$\widetilde{TIC} = \widetilde{TIC}_r + \widetilde{TIC}_p$$

$$= \frac{1}{\tilde{C}}\left[\tilde{S}_m + \sum_{j=1}^{l}\frac{\tilde{S}_{rj}}{n_j}\right]$$

$$+ \frac{\tilde{C}}{2}\left[\widetilde{H}_m\sum_{i=1}^{m}\frac{\widetilde{D}_i^{\,2}}{\tilde{P}} + \sum_{j=1}^{l}\widetilde{M}_j\widetilde{H}_{rj}\sum_{i=1}^{m}\widetilde{D}_i\left(n_j - 1 + \frac{\sum_{i=1}^{m}\widetilde{D}_i}{\tilde{P}}\right)\right] \quad \ldots\ldots (8)$$

Fuzzy Revenue of Manufacturer is $\sum_{i=1}^{m} \widetilde{D}_i \tilde{P}_m$.

Fuzzy Net profit of the Manufacturer is,

$$\widetilde{NP}_m(\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_m, \tilde{n}_1, \tilde{n}_2, \dots, \tilde{n}_l, \tilde{C}, \tilde{A}, \tilde{P}_m) = \sum_{i=1}^{m} \widetilde{D}_i \tilde{P}_m - \widetilde{TDC}_p - \widetilde{TIDC}_p \quad \dots (9)$$

3.2.1. Stackelberg game model

Fuzzy net profit functions for retailer i and the manufacturer in equations (1) and (9) respectively. The pricing advertising and inventory decision model as a stackelberg game model:

$$\max \widetilde{NP}_m(\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_m, \tilde{n}_1, \tilde{n}_2, \dots, \tilde{n}_l, \tilde{C}, \tilde{A}, \tilde{P}_m) = \sum_{i=1}^{m} \widetilde{D}_i \tilde{P}_m - \widetilde{TDC}_p - \widetilde{TIDC}_p \quad \dots\dots (10)$$

Subject to

$$\sum_{i=1}^{m} \widetilde{D}_i \leq \tilde{P} \quad \dots\dots (11)$$

$$0 \leq \tilde{b}_i \leq 1, i = 1,2, \dots m \quad \dots\dots (12)$$

$$n_j, j = 1,2, \dots l \ are\ interger\ variables \quad \dots\dots (13)$$

$$\tilde{C}, \tilde{A}, \tilde{P}_m \geq 0 \quad \dots\dots (14)$$

$$max\ \widetilde{NP}_{ri}(\tilde{P}_{ri}, \tilde{a}_i) = (\tilde{P}_{ri} - \tilde{P}_m - \gamma_i)\widetilde{D}_i - \tilde{a}_i, i = 1,2, \dots m \quad \dots\dots (15)$$

Subject to

$$\tilde{P}_{ri} > \tilde{P}_m + \gamma_i, i = 1,2, \dots m \quad \dots\dots (16)$$

$$\tilde{P}_{ri}, \tilde{a}_i \geq 0, i = 1,2, \dots m \quad \dots\dots (17)$$

Eqs. (10) and (15) are the objective functions of the manufacturer and its retailers, respectively; Constraint (11) is the production capacity constraint; Constraints (12) shows that each retailer's backorder fraction is between 0 and 1; Constraints (17) represent the basic existing condition of the retailers. The decision variables are defined by the remaining constraints.

3.2.2. Retailer's best response functions:

First derivative of equation (15) with respect to $\tilde{a}_i$,

$$\frac{\partial}{\partial \tilde{a}_i} \widetilde{NP}_{ri}(\tilde{P}_{ri}, \tilde{a}_i) = (\tilde{P}_{ri} - \tilde{P}_m - \gamma_i)\frac{\partial \widetilde{D}_i}{\partial \tilde{a}_i} - 1, i = 1,2, \dots m \quad \dots\dots (18)$$

$Solving\ \frac{\partial}{\partial \tilde{a}_i}\ \widetilde{NP}_{ri}(\tilde{P}_{ri}, \tilde{a}_i) = 0$,the critical point of the equation with $\widetilde{D}_i\ and\ \tilde{a}_i. Because\ \frac{\partial^2}{\partial \tilde{a}_i^{\ 2}}\ \widetilde{NP}_{ri}(\tilde{P}_{ri}, \tilde{a}_i) < 0$.

Only one critical point exists and it is a function of $(\tilde{P}_{ri}, \tilde{a}_i)$. the critical point is the optimal solution of retailer I for any given $(\tilde{P}_{ri}, \tilde{a}_i)$ and denoted as,

$$\tilde{a}_i^{\ *} = \tilde{a}_i^{\ *}(\tilde{P}_{ri}, \tilde{A}) \quad \ldots\ldots (19)$$

First derivative of $\widetilde{NP}_{ri}(\tilde{P}_{ri}, \tilde{a}_i)$with respect to $\tilde{P}_{ri}$,

$$\frac{\partial}{\partial \tilde{P}_{ri}}\ \widetilde{NP}_{ri}(\tilde{P}_{ri}, \tilde{a}_i) = \widetilde{D}_i + (\tilde{P}_{ri} - \tilde{P}_m - \gamma_i) \left(\frac{\partial \widetilde{D}_i}{\partial \tilde{P}_{ri}} + \frac{\partial \widetilde{D}_i}{\partial \tilde{a}_i}\frac{\partial \tilde{a}_i^{\ *}}{\partial \tilde{P}_{ri}}\right) - 1\ , i = 1,2, \ldots m \quad \ldots\ldots (20)$$

Equations (19) and (20) are the best response functions of retailer i, with a given specific demand function $\widetilde{D}_i$, which will be considered as the constraints in the manufacturer's decision process.

3.2.3. Manufacturer's decision problem:

The manufacturer determines its optimal the common replenishment cycle $\tilde{C}$, wholesale price $\tilde{P}_m$, advertising cost $\tilde{A}$ and backorder fraction $\tilde{b}_i$for the finished product and the replenishment cycle factor $\tilde{n}_j$, for all raw materials to maximize its own net profit subject to the constraints imposed by equations (11) to (14) and considering the retailer's best response above. Manufacturer's problem is,

$$max\ \widetilde{NP}_m(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_m, \tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_l, \tilde{C}, \tilde{A}, \tilde{P}_m) = \sum_{i=1}^{m} \widetilde{D}_i \tilde{P}_m - \widetilde{TDC}_p - \widetilde{TIDC}_p$$

$$= \sum_{i=1}^{m} \widetilde{D}_i \tilde{P}_m - \sum_{i=1}^{m} \widetilde{D}_i \left(\tilde{C}_m + \phi_i + \sum_{j=1}^{l} \widetilde{M}_j \tilde{C}_{rj}\right)$$

$$- \sum_{i=1}^{m} \left(\frac{\tilde{S}_{bi}}{\tilde{C}} + \frac{\widetilde{D}_i(1-\tilde{b}_i)^2 \tilde{C}}{2} \widetilde{H}_{bi} + \frac{\widetilde{D}_i \tilde{b}_i^2 \tilde{C}}{2} \tilde{L}_{ri} - \gamma_i \widetilde{D}_i\right) - \frac{1}{\tilde{C}}\left[\tilde{S}_m + \sum_{j=1}^{l} \frac{\tilde{S}_{rj}}{n_j}\right]$$

$$+ \frac{\tilde{C}}{2}\left[\widetilde{H}_m \sum_{i=1}^{m} \frac{\widetilde{D}_i^{\ 2}}{\tilde{P}} + \sum_{j=1}^{l} \widetilde{M}_j \widetilde{H}_{rj} \sum_{i=1}^{m} \widetilde{D}_i \left(n_j - 1 + \frac{\sum_{i=1}^{m} \widetilde{D}_i}{\tilde{P}}\right)\right] - \tilde{A} \quad \ldots\ldots (21)$$

$$\frac{\partial}{\partial \tilde{b}_i^{\,2}}\ \widetilde{NP}_m\left(\tilde{b}_1,\tilde{b}_2,\dots,\tilde{b}_m,\tilde{n}_1,\tilde{n}_2,\dots,\tilde{n}_l,\tilde{C},\tilde{A},\tilde{P}_m\right) = -\,\tilde{C}.\widetilde{D}_i\left(\widetilde{H}_{bi}+\tilde{L}_{ri}\right) < 0.$$

First derivative of equation (21) with respect to

$$\tilde{b}_i,\ \tilde{b}_i = \frac{\widetilde{H}_{bi}}{\widetilde{H}_{bi}+\tilde{L}_{ri}}\ ,i = 1,2,\dots m \quad \dots\dots (22)$$

By substituting equation (22) into equation (21),

$$\widetilde{NP}_m\left(\tilde{n}_1,\tilde{n}_2,\dots,\tilde{n}_l,\tilde{C},\tilde{A},\tilde{P}_m\right)$$

$$= \sum_{i=1}^{m}\widetilde{D}_i(\tilde{P}_m+\tilde{\gamma}_i) - \sum_{i=1}^{m}\widetilde{D}_i\left(\tilde{C}_m+\tilde{\phi}_i+\sum_{j=1}^{l}\widetilde{M}_j\tilde{C}_{rj}\right) - \frac{1}{\tilde{C}}\left[\tilde{S}_m+\sum_{i=1}^{m}\tilde{S}_{bi}+\sum_{j=1}^{l}\frac{\tilde{S}_{rj}}{\tilde{n}_j}\right]$$

$$+\sum_{i=1}^{m}\frac{\widetilde{D}_i\widetilde{H}_{bi}\tilde{L}_{ri}}{\widetilde{H}_{bi}+\tilde{L}_{ri}} - \frac{\tilde{C}}{2}\widetilde{H}_m\sum_{i=1}^{m}\frac{\widetilde{D}_i^{\,2}}{\tilde{P}}$$

$$+\sum_{j=1}^{l}\widetilde{M}_j\widetilde{H}_{rj}\sum_{i=1}^{m}\widetilde{D}_i\left(n_j-1+\frac{\sum_{i=1}^{m}\widetilde{D}_i}{\tilde{P}}\right) - \tilde{A} \quad \dots\dots (23)$$

$$\frac{\partial}{\partial \tilde{C}^2}\widetilde{NP}_m\left(\tilde{n}_1,\tilde{n}_2,\dots,\tilde{n}_l,\tilde{C},\tilde{A},\tilde{P}_m\right) = -\frac{2}{\tilde{C}^3}\left[\tilde{S}_m+\sum_{i=1}^{m}\tilde{S}_{bi}+\sum_{j=1}^{l}\frac{\tilde{S}_{rj}}{\tilde{n}_j}\right] < 0$$

$$\frac{\partial}{\partial \tilde{C}}\widetilde{NP}_m\left(\tilde{n}_1,\tilde{n}_2,\dots,\tilde{n}_l,\tilde{C},\tilde{A},\tilde{P}_m\right) = 0, \textit{the optimal value of } \tilde{C} \textit{ is}$$

$$\tilde{C} = \sqrt{\frac{2\left(\tilde{S}_m+\sum_{i=1}^{m}\tilde{S}_{bi}+\sum_{j=1}^{l}\frac{\tilde{S}_{rj}}{\tilde{n}_j}\right)}{\widetilde{H}\sum_{i=1}^{m}\widetilde{D}_i}} \quad \dots\dots (24)$$

$$\widetilde{H} = \frac{1}{\sum_{i=1}^{m}\widetilde{D}_i}\left[\widetilde{H}_m\sum_{i=1}^{m}\frac{\widetilde{D}_i^{\,2}}{\tilde{P}} - \sum_{i=1}^{m}\frac{\widetilde{D}_i\widetilde{H}_{bi}\tilde{L}_{ri}}{\widetilde{H}_{bi}+\tilde{L}_{ri}} + \sum_{j=1}^{l}\widetilde{M}_j\widetilde{H}_{rj}\left(n_j-1+\frac{\sum_{i=1}^{m}\widetilde{D}_i}{\tilde{P}}\right)\right]$$

By substituting equation (24) into equation (23),

$$\widetilde{NP}_m\left(\tilde{A},\tilde{P}_m\right) = \sum_{i=1}^{m}\widetilde{D}_i(\tilde{P}_m+\tilde{\gamma}_i) - \sum_{i=1}^{m}\widetilde{D}_i\left(\tilde{C}_m+\tilde{\phi}_i+\sum_{j=1}^{l}\widetilde{M}_j\tilde{C}_{rj}\right)$$

$$-\tilde{A} - \sqrt{2\widetilde{H}\left(\tilde{S}_m + \sum_{i=1}^{m} \tilde{S}_{bi} + \sum_{j=1}^{l} \frac{\tilde{S}_{rj}}{\tilde{n}_j}\right)} \dots\dots (25)$$

Subject to $\sum_{i=1}^{m} \widetilde{D}_i \leq \tilde{P}$ and $\tilde{A}, \tilde{P}_m \geq 0$

Here only two variables are left, and they are continuous Kuhn-Tucker condition can be used to calculate the optimal solutions. Let λ be Lagrange multiplier, Lagrange function L_m can expressed as,

$$max\ \tilde{L}_m(\tilde{P}_m, \tilde{A}, \lambda) = \widetilde{N}p_m(\tilde{P}_m, \tilde{A}) + \lambda\left(\widetilde{P} - \sum_{i=1}^{m} \widetilde{D}_i\right)$$

Then the KKT conditions for the model is follows,

$$\frac{\partial \widetilde{N}p_m(\tilde{P}_m, \tilde{A})}{\partial \tilde{P}_m} - \lambda \frac{\sum_{i=1}^{m} \widetilde{D}_i}{\partial \tilde{P}_m} = 0, \frac{\partial \widetilde{N}p_m(\tilde{P}_m, \tilde{A})}{\partial \tilde{A}} - \lambda \frac{\sum_{i=1}^{m} \widetilde{D}_i}{\partial \tilde{A}} = 0,$$

$$\lambda\left(\widetilde{P} - \sum_{i=1}^{m} \widetilde{D}_i\right) = 0$$

$For\ \lambda = 0$, we have $\frac{\partial \widetilde{N}p_m(\tilde{P}_m, \tilde{A})}{\partial \tilde{P}_m} = 0, \frac{\partial \widetilde{N}p_m(\tilde{P}_m, \tilde{A})}{\partial \tilde{A}} = 0.$

And give the corresponding critical point $\tilde{P}_m\ and\ \tilde{A}$.

3.2.4. Stackelberg game equilibrium:

$from\ equation$ (15), $\widetilde{NP}_{ri}(\tilde{P}_{ri}, \tilde{a}_i) = (\tilde{P}_{ri} - \tilde{P}_m - \tilde{\gamma}_i)\widetilde{D}_i - \tilde{a}_i$

By using cobb-douglas demand function,

$$\widetilde{NP}_{ri}(\tilde{P}_{ri}, \tilde{a}_i) = (\tilde{P}_{ri} - \tilde{P}_m - \tilde{\gamma}_i)K_i \frac{\tilde{a}_i^{\alpha_i} \widetilde{A}^{\beta_i}}{\widetilde{P}_{ri}^{\rho_i}} - \tilde{a}_i$$

$$\frac{\partial}{\partial \tilde{a}_i} \widetilde{NP}_{ri}(\tilde{P}_{ri}, \tilde{a}_i) = \alpha_i(\tilde{P}_{ri} - \tilde{P}_m - \tilde{\gamma}_i)K_i \frac{\tilde{a}_i^{\alpha_i - 1} \widetilde{A}^{\beta_i}}{\widetilde{P}_{ri}^{\rho_i}} - 1 = 0$$

$$\tilde{a}_i^* = \left[\alpha_i(\tilde{P}_{ri} - \tilde{P}_m - \tilde{\gamma}_i)K_i \frac{\widetilde{A}^{\beta_i}}{\widetilde{P}_{ri}^{\rho_i}}\right]^{\frac{1}{1-\alpha_i}}$$

The optimal value of $\tilde{P}_{ri}$, $\tilde{P}_{ri}^{\ *} = \frac{(C_p+\gamma_i)\rho_i}{\rho_i - 1}, i = 1,2 \dots m.$

4. Numerical example

The primary purpose of this numerical example is to demonstrate the results of the proposed Stackelberg game and its solution algorithm, meaningful game parameters should be reasonably set. This is done by carefully investigating suggestions and practices given by other researchers [02,05,09], and some properties of the Cobb–Douglas function originally used in supply chain practices. The holding cost per unit finished product at a retailer side should be higher than in the manufacturer's side. The backorder cost per unit product should be higher than holding cost per unit product. After careful considerations, values for input parameters of the example are given in Table. As an illustration, the case of 3 retailers and two kinds of raw materials are discussed. The unit time is one year. From manufacturer's point of view, we can understand that having VMI policy and offering uniform pricing is the best strategy, while from retailer's point of view, the independent periodic inventory policy is the best policy.

	Retailer 1	Retailer 2	Retailer 3
Fuzzy Backorder cost $\tilde{L}_{bi}$	(300,400,500)	(400,500,600)	(500,600,700)
Fuzzy holding cost $\tilde{H}_{bi}$	(10,11,12)	(11,12,13)	(12,13,14)
Fuzzy Transportation Cost ϕ_i	(8,9,10)	(9,10,11)	(10,11,12)
Fuzzy Inventory cost $\tilde{\gamma}_i$	(6,7,8)	(7,8,9)	(8,9,10)
Fuzzy Order cost $\tilde{S}_{bi}$	(80,90,100)	(90,100,110)	(100,110,120)

m=3 l=2

$K_1 = 300$ $K_2 = 350$ $K_3 = 400$ $\alpha_1=0.41$ $\alpha_2=0.43$ $\alpha_3 = 0.45$

$\beta_1 = 0.37$ $\beta_2 = 0.39$ $\beta_3 = 0.41$ $\rho_1 = 1.2$ $\rho_2 = 1.3$ $\rho_3 = 1.4$

$\tilde{P} = (49000,50000,51000)$ $\tilde{A} = (0.64 \times 10^6, 0.65 \times 10^6, 0.66 \times 10^6)$

$\tilde{S}_p = (100,200,300)$ $\tilde{C}_p = (150,200,250)$ $\tilde{H}_p = (3,4,5)$

$\tilde{S}_{r1} = (300,400,500)$ $\tilde{S}_{r2} = (400,500,600)$ $\tilde{C}_m = (15,20,25)$

$\tilde{H}_{r1} = (1,1.5,2)$ $\tilde{H}_{r2} = (1.5,2,2.5)$ $n_1 = 2$ $n_2 = 2$

$\tilde{C}_{r1} = (10,15,20)$ $\tilde{C}_{r2} = (15,20,25)$

4.1. Fraction of Fuzzy backlogging rate in a cycle for retailer i

$$\tilde{b}_1^* = \frac{\tilde{H}_{b1}}{\tilde{H}_{b1} + \tilde{L}_{b1}} = \frac{(10,11,12)}{(10,11,12) + (300,400,500)} = \frac{(10,11,12)}{(310,411,512)}$$

$$\tilde{b}_1^* = (0.0195, 0.0268, 0.0387)$$

Similarly,

$$\tilde{b}_2^* = (0.0179, 0.0234, 0.0316)$$

$$\tilde{b}_3^* = (0.0168, 0.0212, 0.0273)$$

4.2. Fuzzy Retail Price

$$P_{r1} = \frac{(C_p + \gamma_1)\rho_1}{\rho_1 - 1} = \frac{[(150, 200, 250) + (6,7,8)]\,(1.2)}{(0.2)} = (936, 1242, 1548)$$

Similarly,

$$P_{r2} = (680, 901, 1122)$$

$$P_{r3} = (553, 732, 910)$$

4.3. Fuzzy Advertising cost

$$\tilde{a}_1{}^* = \left[\alpha_1\left(\tilde{P}_{r1} - \tilde{P}_m - \tilde{\gamma}_1\right)K_1\tilde{A}^{\beta_1}\right]^{\frac{1}{1-\alpha_1}} \tilde{P}_{r1}^{\frac{-\rho_1}{1-\alpha_1}}$$

$$= \left[\begin{array}{c}\{(936\,,1242,1548) - (156,207,258)\} \\ (300)(0.41)(0.64 \times 10^6, 0.65 \times 10^6, 0.66 \times 10^6)^{0.37}\end{array}\right]^{\frac{1}{0.59}}$$

$$\div\, [(936,1242,1548)]^{\frac{1.2}{0.59}}$$

$$= (312178.072, 1010518.613, 2996996.723)$$

$$\tilde{a}_1{}^* = (0.3122\ X10^6\ , 1.0105\ X10^6\ , 2.9970\ X10^6\ )$$

Similarly,

$$\tilde{a}_2{}^* = (0.2759\ X10^6\ , 1.1025\ X10^6\ , 3.7839\ X10^6\ )$$

$$\tilde{a}_3{}^* = (0.2409\ X10^6\ , 1.2162\ X10^6\ , 4.8608\ X10^6\ )$$

4.4. Fuzzy Demand rate of the finished product

$$\widetilde{D}_1 = K_1 \frac{\tilde{a}_1^{\alpha_1}\, \widetilde{A}^{\beta_1}}{\widetilde{P}_{r1}^{\rho_1}}$$

$$= \frac{300\,(0.3122\,X10^6\,,1.0105\,X10^6\,,2.9970\,X10^6)^{0.43}\,(0.64\times10^6, 0.65\times10^6, 0.66\times10^6)^{0.39}}{(936\,,1242,1548)^{1.3}}$$

$$\widetilde{D}_1 = (1123.0528, 2381.2948, 5251.2544) = (1123, 2381, 5251)$$

Similarly,

$$\widetilde{D}_2 = (1524, 3700, 9119)$$

$$\widetilde{D}_3 = (1827, 5168, 14364)$$

4.5. Fuzzy Common replenishment cycle time for the finished product

$$\widetilde{H} = \frac{1}{\sum_{i=1}^{m}\widetilde{D}_i}\left[\widetilde{H}_m \sum_{i=1}^{m}\frac{\widetilde{D}_i^{\,2}}{\widetilde{P}} - \sum_{i=1}^{m}\frac{\widetilde{D}_i\widetilde{H}_{bi}\widetilde{L}_{ri}}{\widetilde{H}_{bi}+\widetilde{L}_{ri}} + \sum_{j=1}^{l}\widetilde{M}_j\widetilde{H}_{rj}\left(n_j - 1 + \frac{\sum_{i=1}^{m}\widetilde{D}_i}{\widetilde{P}}\right)\right]$$

$$\sum_{i=1}^{m}\widetilde{D}_i = (4474, 11249, 28734)$$

$$\frac{\widetilde{H}_m}{P}\left[\widetilde{D}_1^2 + \widetilde{D}_2^2 + \widetilde{D}_3^2\right]$$

$$= \frac{(3,4,5)}{(49000,50000,51000)}\left[\begin{array}{c}(1123, 2381, 5251)^2 + (1524, 3700, 9119)^2\\ +(1827, 5168, 14364)^2\end{array}\right]$$

$$= (407.1549, 3685.3908, 32352.4141)$$

$$\sum_{i=1}^{m}\frac{\widetilde{D}_i\widetilde{H}_{bi}\widetilde{L}_{ri}}{\widetilde{H}_{bi}+\widetilde{L}_{ri}} = \left\{\begin{array}{l}\frac{(1123, 2381, 5251)(300,400,500)(10,11,12)}{(310,411,512)}\\ +\frac{(1524, 3700, 9119)(400,500,600)(11,12,13)}{(411,512,613)}\\ +\frac{(1827, 5168, 14364)(500,600,700)(12,13,14)}{(512,613,714)}\end{array}\right\}$$

$$= (32872.0079, 134608.967, 549629.5095)$$

$$\left(n_j - 1 + \frac{\sum_{i=1}^{m}\widetilde{D}_i}{\widetilde{P}}\right) = 1 + \frac{(4474, 11249, 28734)}{(49000, 50000, 51000)} = (1.0877, 1.2250, 1.5864)$$

$$\sum_{j=1}^{l}\widetilde{M}_j\widetilde{H}_{rj}\left(n_j - 1 + \frac{\sum_{i=1}^{m}\widetilde{D}_i}{\widetilde{P}}\right) = (0.9, 1, 1.1)(1, 1.5, 2)(1.0877, 1.225, 1.5864)$$

$$+(1, 1.1, 1.2)(1.5, 2, 2.5)(1.0877, 1.225, 1.5864)$$

$$= (2.6105, 4.5325, 8.2493)$$

$$\tilde{H} = \frac{1}{(4474, 11249, 28734)}[(407.1549, 3685.3908, 32352.4141) + (32872.0079, 134608.967, 549629.5095) + (2.6105, 4.5325, 8.2493)]$$

$$\tilde{H} = (1.1583, 12.2943, 130.0827)$$

$$\tilde{C} = \sqrt{\frac{2\left(\tilde{S}_m + \sum_{i=1}^{m}\tilde{S}_{bi} + \sum_{j=1}^{l}\frac{\tilde{S}_{rj}}{\tilde{n}_j}\right)}{\tilde{H}\sum_{i=1}^{m}\tilde{D}_i}} = \sqrt{\frac{2\left(\tilde{S}_m + \tilde{S}_{b1} + \tilde{S}_{b2} + \tilde{S}_{b3} + \frac{\tilde{S}_{r1}}{\tilde{n}_1} + \frac{\tilde{S}_{r2}}{\tilde{n}_2}\right)}{\tilde{H}\sum_{i=1}^{3}\tilde{D}_i}}$$

$$= \sqrt{\frac{2[(100, 200, 300) + (270, 300, 330) + (150, 200, 250) + (200, 250, 300)]}{(1.1583, 12.2943, 130.0827)(4474, 11249, 28734)}}$$

$$\tilde{C} = (0.02, 0.1170, 0.6748)$$

4.6. Fuzzy Net profit for retailer i ($\widetilde{NP}_{ri}$)

$$\widetilde{NP}_{r1} = (\tilde{P}_{r1} - \tilde{P}_m - \gamma_1)\,\tilde{D}_1 - \tilde{a}_1$$

$$\widetilde{NP}_{r1} = [(936, 1242, 1548) - (150, 200, 250) - (6, 7, 8)]\,(1123, 2381, 5251) - (0.3122\,X10^6, 1.0105\,X10^6, 2.9970\,X10^6)$$

$$\widetilde{NP}_{r1} = (-2235606, 1453835, 6997192)$$

Similarly,

$$\widetilde{NP}_{r2} = (-3142296, 14616100, 8523935)$$

$$\widetilde{NP}_{r3} = (-4325489, 1486664, 10560828)$$

4.7. Fuzzy total direct cost for the finished product ($\widetilde{TDC}_p$)

$$\widetilde{TDC}_p = \sum_{i=1}^{m}\tilde{D}_i\left(\tilde{C}_m + \phi_i + \sum_{j=1}^{l}\tilde{M}_j\tilde{C}_{rj}\right)$$

$$\sum_{j=1}^{l}\tilde{M}_j\tilde{C}_{rj} = (0.9, 1, 1.1)(10, 15, 20) + (1, 1.1, 1.2)(15, 20, 25) = (24,37,52)$$

$$\widetilde{TDC}_p = \left\{\begin{array}{c}(1123, 2381, 5251)[(15, 20, 25) + (8, 9, 10) + (24,37,52)] \\ + (1524, 3700, 9119)[(15, 20, 25) + (9, 10, 11) + (24,37,52)] \\ + (1827, 5168, 14364)[(15, 20, 25) + (10, 11, 12) + (24,37,52)]\end{array}\right\}$$

$$\widetilde{TDC}_p = (215456, 756470, 2537705)$$

4.8. Compensation cost for retailer i ($\widetilde{CC}_i$)

$$\widetilde{CC}_1 = \frac{\tilde{S}_{b1}}{\tilde{C}} + \frac{\widetilde{D}_1(1-\tilde{b}_1)^2\tilde{C}}{2}\widetilde{H}_{b1} + \frac{\widetilde{D}_1\tilde{b}_1^2\tilde{C}}{2}\tilde{L}_{r1} - \gamma_1\widetilde{D}_1$$

$$\frac{\tilde{S}_{b1}}{\tilde{C}} = \frac{(80, 90, 100)}{(0.02, 0.1170, 0.6748)} = (118.5536, 769.2308, 5000)$$

$$\frac{\widetilde{D}_1\tilde{C}}{2}\left[\left(1-\tilde{b}_1\right)^2\widetilde{H}_{b1} + \tilde{b}_1^2\tilde{L}_{r1}\right] = \frac{1}{2}\ (1123, 2381, 5251)(0.02, 0.1170, 0.6748)$$

$$\{[1-(0.0195, 0.0268, 0.0387)]^2(10, 11, 12) + (0.0195, 0.0268, 0.0387)^2(300, 400, 500)\}$$

$$= (105.0578, 1491.1670, 21765.8884)$$

$$\gamma_1\widetilde{D}_1 = (6, 7, 8)(1123, 2381, 5251) = (6738, 16667, 42008)$$

$$\widetilde{CC}_1 = (-41784.3886, -14406.6022, 20027.8884)$$

Similarly,

$$\widetilde{CC}_2 = (-81778.4621, -26208.7648, 35254.0417)$$

$$\widetilde{CC}_3 = (-143321.7961, -41724.9169, 59501.7971)$$

4.9. Fuzzy total inventory cost for the finished product and raw materials ($\widetilde{TIC}$)

$$\widetilde{TIC} = \frac{1}{\tilde{C}}\left[\tilde{S}_m + \sum_{j=1}^{l}\frac{\tilde{S}_{rj}}{n_j}\right] + \frac{\tilde{C}}{2}\left[\widetilde{H}_m\sum_{i=1}^{m}\frac{\widetilde{D}_i^{\,2}}{\tilde{P}} + \sum_{j=1}^{l}\widetilde{M}_j\widetilde{H}_{rj}\sum_{i=1}^{m}\widetilde{D}_i\left(n_j - 1 + \frac{\sum_{i=1}^{m}\widetilde{D}_i}{\tilde{P}}\right)\right]$$

$$\frac{1}{\tilde{C}}\left[\tilde{S}_m + \sum_{j=1}^{l}\frac{\tilde{S}_{rj}}{n_j}\right] = \frac{[(100, 200, 300) + (150, 200, 250) + (200, 250, 300)]}{(0.02, 0.1170, 0.6748)}$$

$$= (666.8643, 5555.5556, 42500)$$

$$\widetilde{H}_m\sum_{i=1}^{m}\frac{\widetilde{D}_i^{\,2}}{\tilde{P}} = (407.1549, 3685.3908, 32352.4141)$$

$$\widetilde{M}_1\widetilde{H}_{r1}\sum_{i=1}^{m}\widetilde{D}_i\left(n_1 - 1 + \frac{\sum_{i=1}^{m}\widetilde{D}_i}{\tilde{P}}\right) + \widetilde{M}_2\widetilde{H}_{r2}\sum_{i=1}^{m}\widetilde{D}_i\left(n_2 - 1 + \frac{\sum_{i=1}^{m}\widetilde{D}_i}{\tilde{P}}\right)$$

$$= (0.9, 1, 1.1)(1, 1.5, 2)(4474, 11249, 28734)\left[1 + \frac{(4474, 11249, 28734)}{(49000, 50000, 51000)}\right]$$

$$+ (1, 1.1, 1.2)(1.5, 2, 2.5)(4474, 11249, 28734)\left[1 + \frac{(4474, 11249, 28734)}{(49000, 50000, 51000)}\right]$$

$$= (11679.2875, 50986.0925, 237034.8115)$$

$$\frac{\tilde{C}}{2}\left[\widetilde{H}_m\sum_{i=1}^{m}\frac{\widetilde{D}_i^{\,2}}{\tilde{P}} + \sum_{j=1}^{l}\widetilde{M}_j\widetilde{H}_{rj}\sum_{i=1}^{m}\widetilde{D}_i\left(n_j - 1 + \frac{\sum_{i=1}^{m}\widetilde{D}_i}{\tilde{P}}\right)\right]$$

$$= \frac{(0.02, 0.1170, 0.6748)}{2}\begin{bmatrix} (407.1549, 3685.3908, 32352.4141) + \\ (11679.2875, 50986.0925, 237034.8115) \end{bmatrix}$$

$$= (120.8644, 3198.2818, 90891.2499)$$

$$\widetilde{TIC} = (666.8643, 5555.5556, 42500) + (120.8644, 3198.2818, 90891.2499)$$

$$\widetilde{TIC} = (787.7297, 8753.8374, 133391.2499)$$

4.10. Total indirect cost for the manufacturer ($\widetilde{TIDC}_p$)

$$\widetilde{TIDC}_p = \sum_{i=1}^{m} \widetilde{CC}_i + \widetilde{TIC} + \tilde{A}$$

$$\begin{aligned}\widetilde{TIDC}_p = {} & (-41784.3886, -14406.6022, 20027.8884) \\ & + (-81778.4621, -26208.7648, 35254.0417) \\ & + (-143321.7961, -41724.9169, 59501.7971) \\ & + (787.7297, 8753.8374, 133391.2499) \\ & + (0.64 \times 10^6, 0.65 \times 10^6, 0.66 \times 10^6)\end{aligned}$$

$$\widetilde{TIDC}_p = (373903.0819, 576413.5535, 908174.9771)$$

4.11. Fuzzy net profit for the manufacturer ($\widetilde{NP}_m$)

$$\widetilde{NP}_m = \sum_{i=1}^{m} \widetilde{D}_i \tilde{P}_m - \widetilde{TDC}_p - \widetilde{TIDC}_p$$

$$\sum_{i=1}^{m} \widetilde{D}_i \tilde{P}_m = (4474, 11249, 28734)(150, 200, 250) = (671100, 2249800, 7183500)$$

$$\begin{aligned}\widetilde{NP}_m = {} & (671100, 2249800, 7183500) - (215456, 756470, 2537705) \\ & - (373903.0819, 576413.5535, 908174.9771)\end{aligned}$$

$$\widetilde{NP}_m = (-2774779.977, 916916.4465, 6594140.918)$$

Using Graded mean integration representation method, we will get the crisp value of the demand rate of retailers, Net profit for manufacturer and retailers.

	Retailer 1	Retailer 2	Retailer 3
Fraction of backlogging rate (b_i)	0.0276	0.0238	0.0215
Retail price (P_{ri})	Rs. 1242	Rs. 901	Rs. 732
Advertising cost (a_i)	Rs. 1.2252 X 10^6	Rs. 1.4116 X 10^6	Rs. 1.6611 X 10^6
Demand rate (D_i)	2650 units	4241 units	6144 units
Compensation cost (CC_i)	-13230.00	-25226.5799	-41786.6111
Net profit (NP_{ri})	Rs. 1762821.00	Rs. 1871340.00	Rs. 2030333.00

Common replenishment cycle time for the finished product C = 0.1938

Total direct cost for the finished product TDC_p = Rs. 963173.5

Total inventory cost for the finished product and raw materials TIC = Rs. 28199.00

Total indirect cost for the manufacturer $TIDC_p$ = Rs. 597955.00

Net profit for manufacturer NP_m = Rs. 1247838.00

5. Conclusion

In this paper, we developed new fuzzy VMI supply chain problem with one manufacturer and multiple retailers as a Stackelberg game model. A computational algorithm has been proposed to solve this game model based on the theoretical analysis of the best response functions with a generic demand function. Here we find out the total inventory cost for the finished product and raw materials, Net profit for manufacturer and all retailers.

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