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Effects of Heat Generation, Thermal Diffusion, Magnetic Field and Chemical Reaction on Demixing of a Binary Fluid Mixture

B.R. Sharma and Kabita Nath

Department of Mathematics, Dibrugarh University,
Dibrugarh-786004, Assam, India.
E-mail: [1]bishwaramsharma@yahoo.com [2]kabitanath56@gmail.com

Abstract

The present problem concerns with the effects of heat generation / absorption, chemical reaction, suction / injection and thermal diffusion on demixing of a binary mixture of incompressible viscous electrically conducting fluids in two dimensional steady, laminar, magnetohydrodynamics boundary layer flow over a permeable cylinder. The momentum, energy and concentration equations are reduced to non-linear coupled ordinary differential equations by similarity transformations and are solved numerically by using MATLAB's built in solver bvp4c. The local skin friction, the Nusselt number and the Sherwood number are tabulated for various values of the parameters. These numerical results are exhibited graphically from which it is found that the effects of various parameters are to separate the components of the binary mixture by collecting the lighter and rarer component near the surface of the cylinder and throwing the heavier one away from it.

Keywords: Heat generation, chemical reaction, thermal diffusion, magnetic field and binary fluid mixture.

Introduction

Combined heat and mass transfer problems with chemical reaction are importance in many processes and have, therefore, received a considerable amount of attention in recent years. Separation processes of an electrically conducting binary mixture of incompressible viscous fluids under the influence of magnetic field are considered to be of significant importance due to their applications in many engineering problems such as nuclear reactors and those dealing with liquid metals. The demixing of isotopes in its naturally occurring mixture is one such example. The composition of binary mixture is described by the concentration c_1, defined as the ratio of mass of rarer and lighter component to the total mass of the mixture in a given volume. The concentration c_2 of heavier and abundant component is given by c_2 = 1-c_1. A binary mixture subject to the temperature gradient can generate thermal diffusion i.e. the temperature gradients cause solute fluxes. This phenomenon is known as Soret effect. In the flow of such a mixture the diffusion of individual species takes place by three mechanisms namely concentration gradient, pressure gradient and temperature gradient. The diffusion flux $\vec{\imath}$ is given by Landau and Lifshitz [1] as:

$$\vec{\imath} = -\rho D\left[\nabla c_1 + k_p \nabla \mathrm{P} + \mathrm{k_T}\, \nabla \mathrm{T}\right] \tag{1}$$

Where ρ is the density of the fluid, $k_p D$ is the pressure diffusion coefficient and $k_T D$ is the thermal diffusion coefficient. The ordinary diffusion contribution to the mass flux is seen to depend in a complicated way on the concentration gradient of the components present in the mixture. The baro diffusion indicates that there may be a net movement of the components in a mixture if there is a pressure gradient imposed on the system. An example of barodiffusion

[2] is the process of diffusion in the binary mixture of different kinds of gases present in the atmosphere. By reasons of variation of forces of gravity with height thereby causing a density gradient, different constituents of the atmosphere tend to separate out. The pressure gradient created by the gravity as well as the rotation of the earth separates various components of air. The tendency for a mixture to separate under a pressure gradient is very small but use is made of this effect in centrifuge separations in which tremendous pressure gradient is established. Thermal diffusion describes the tendency for species to diffuse under the influence of a temperature gradient. In many practical problems dealing with flows in porous media one encounters with a multiple component electrically conducting fluids, *e.g.*, molten fluid in the earth's crust, crude oil in the petroleum. It is customary to consider one of the components as solvent and the other components as solute. It is shown [3] that if separation due to thermal diffusion occurs then it may even render an unstable system to stable one. This effect is also quite small, but devices can be arranged to produce very steep temperature gradients so that separations of mixtures are affected.

Many researchers have discussed the problem of a stretched surface moving with a linear velocity and various thermal boundary conditions (see, for instance, Crane [4], Gupta and Gupta [5], Vleggar [6], Soundalgekar and Murty [7], Grubka and Bobba [8], and Chen and Char [9]). Later on, many investigators studied the consequent flow and heat transfer characteristics that are brought about by the movement of a stretched permeable and impermeable, isothermal and non-isothermal surface with a power-law velocity variation. For examples are Banks [10] who considered the case of impermeable wall, Ali [11, 12] who presented various extensions to Banks' problem, in terms of flow and thermal boundary conditions. Mahapatra and Gupta [13] studied heat transfer in stagnation-point flow towards a stretching sheet. Pop et al. [14] investigated radiation effects on the flow near the stagnation point of a stretching sheet. Chamkha [15] studied heat and mass transfer from MHD flow over a moving permeable cylinder. Sharma and Singh [16, 17, 18], Sharma and Nath [19, 20] and Sharma et al. [21, 22] have studied the effect of magnetic field on demixing of a binary fluid mixture. Sharma and Singh [23, 24] have studied the effect of temperature gradient on demixing of species in hydromagnetic flow of a binary mixture of incompressible viscous fluids between two parallel plates, first taking the plates horizontal and second by taking the plates vertical. They found that the effect of temperature gradient is to separate the components of the binary mixture and the magnetic field increases the effect of species demixing.

The present paper deals with the two dimensional steady, laminar, MHD boundary layer flow of an incompressible viscous electrically conducting binary mixture of fluids flowing over a moving permeable cylinder with heat generation, chemical reaction, thermal diffusion and magnetic field effects in the presence of uniform transverse magnetic field.

Formulation of the Problem

Consider steady, laminar, hydromagnetic, two-dimensional boundary layer flow, and heat and mass transfer in a binary mixture of incompressible viscous electrically conducting fluids driven by a vertical isothermal permeable cylinder moving with a linear velocity in the fluid mixture infinite in extent with heat generation / absorption, chemical reaction and thermal diffusion effects. Fluid suction or injection is imposed at the boundary of the cylinder and a uniform transverse weak magnetic field is applied normal to the flow direction. The external or free stream velocity is assumed to vary linearly with the axial distance. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. In addition, no external electric field is assumed to exist and the electric field due to the effect of charge polarization and the Hall effect of magnetohydrodynamics are neglected. The effects of viscous dissipation and Joule heating are also neglected. The basic equations for this investigation can be written as:

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + u_e(x)\frac{du_e(x)}{dx} - \frac{\sigma B^2}{\rho}\left[u_e(x) - u\right] \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{Q}{\rho c_p}(T - T_\infty) \tag{4}$$

$$u\frac{\partial c_1}{\partial x} + v\frac{\partial c_1}{\partial r} = D\left[\frac{\partial^2 c_1}{\partial r^2} + \frac{1}{r}\frac{\partial c_1}{\partial r} + S_T\left\{c_1\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \left(\frac{\partial c_1}{\partial r}\right)\left(\frac{\partial T}{\partial r}\right)\right\}\right] - r_c(c_1 - c_\infty) \tag{5}$$

Where x and r are the axial and the radial distances, respectively. u, v, T, c_1, T_∞ and c_∞ are the fluid x-component and r-component of velocity, temperature, concentration, respectively. ρ, ν, c_p, α and D are the fluid density, kinematic viscosity, specific heat at constant pressure, thermal diffusivity, and the mass diffusion respectively. σ, B, Q, S_T and r_c are the electrical conductivity, magnetic induction, heat generation /absorption coefficient, the Soret number and the chemical reaction coefficient, respectively. Here $u_e(x)$ is the variable external or free stream velocity. It should be mentioned that positive values of Q mean heat generation (source) and negative values of Q indicate heat absorption (sink). Also, it is noticed that the heat generation or absorption term is assumed to be dependent on the difference between the boundary-layer temperature and that of the free stream. This has been employed previously by Vajravelu and Hadjinicolaou [25] and is appropriate for boundary–layer applications in which this difference is significant.

The boundary conditions for this problem are:

$$u = u_w x, v = -v_w, T = T_w, c_1 = c_w \text{ at } r = R$$
$$u \to u_e(x) = u_\infty x, T \to T_\infty, c_1 \to c_\infty \text{ as } r \to \infty \tag{6}$$

where u_w is a constant and v_w, T_w and c_w are the suction (> 0) or injection (< 0) velocity, wall temperature and wall concentration and R is the radius of the cylinder, respectively.

It is convenient to employ the following similarity transformations:

$$\eta = \sqrt{\frac{u_w}{2\,\nu}}\left(\frac{r^2-R^2}{R}\right), \psi = \sqrt{\frac{\nu u_w}{2}}\, Rxf(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}$$
$$C(\eta) = \frac{c_1-c_\infty}{c_w-c_\infty}, u = \frac{1}{r}\frac{\partial\psi}{\partial r}, v = -\frac{1}{r}\frac{\partial\psi}{\partial x} \tag{7}$$

where η , $f'(\eta)$, $\theta(\eta)$ and $C(\eta)$ are the similarity variable, dimensionless velocity, dimensionless temperature and the dimensionless concentration, respectively. Here, a prime denotes ordinary differentiation with respect to η. It can easily be verified that the balance of mass given by equation (2) is identically satisfied.

The set of equations (3)-(5) are coupled non-linear partial differential equations. Introducing the relation (7) into the equations (3)-(5) we obtain the following non-linear coupled ordinary differential equations

$$(2K\eta + 2)f''' + (2K + f)f'' - (f')^2 - Ha^2(\lambda - f') + \lambda^2 = 0 \tag{8}$$
$$(2K\eta + 2)\theta'' + (2K + \Pr f)\ \theta' + \Pr\ \phi\,\theta = 0 \tag{9}$$
$$(2K\eta + 2)C'' + (2K + S_c f)C' + t_d(2K\eta + 2)\theta''$$
$$+ t_d\{2K + (2K\eta + 2)C'\}\theta' - \gamma S_c C = 0 \tag{10}$$

where $K = \frac{1}{R}\sqrt{\frac{2\nu}{u_w}}$ is the curvature parameter, $Ha = \sqrt{\frac{\sigma B^2}{\rho u_w}}$ is the Hartmann number, $\lambda = \frac{u_\infty}{u_w}$ is the ratio of the free stream to cylinder stretching velocity, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $\phi = \frac{Q}{\rho c_p u_w}$ is the dimensionless heat generation / absorption coefficient which indicates heat generation for positive values and heat absorption for negative values, $S_c = \frac{\nu}{D}$ is the Schmidt number, $\gamma = \frac{r_c}{u_w}$ is the dimensionless chemical reaction parameter and $t_d = S_T(T_w - T_\infty)$ is the Thermal diffusion number.

The transformed dimensionless boundary conditions become:

$$f' = 1, f = f_0, \theta = 1, C = 1 \text{ at } \eta = 0$$
$$f' \to \lambda, \theta \to 0, C \to 0 \text{ as } \eta \to \infty \tag{11}$$

where $f_0 = v_w\sqrt{\frac{2}{\nu\, u_w}}$ is the dimensionless suction ($f_0 > 0$) or injection (($f_0 < 0$) velocity.

Since the solutions of non-linear coupled ordinary differential equations (8) to (10) under boundary conditions (11) cannot be obtained in closed form therefore these equations are solved numerically with MATLAB's built-in solver bvp4c.

Results and Discussion

Numerical calculations have been carried out for various values of the parameters K, γ, f_0, t_d, S_c and λ and these numerical results for concentration of the lighter and rarer component of the binary fluid mixture are plotted against η for various values of above mentioned parameters and are displayed in Figures 1-6. It is observed from the figures that the concentration of the lighter and rarer component of the binary mixture is more near the surface of the cylinder and decreases exponentially as η increases from 0 to 7.

Figure 1 depicts that the concentration of the lighter and rarer component of the binary mixture increases with increase in the values of the parameter K and from Figures 2-6 reverse effect is observed with the increase in the values of the parameters γ, f_0, t_d, S_c and λ.

Thus we conclude that the effects of all these parameters is to demix the binary mixture by collecting the rarer and lighter component of the binary fluid mixture near the surface of the cylinder and to throw the heavier component away from it. From the process of numerical computation, the local skin friction C_f , the Nusselt number Nu and the Sherwood number Sh are also worked out and their numerical values are presented in a tabular form. These are defined, respectively, as follows:

$$C_f = \frac{-\frac{\partial u}{\partial r}(x,R)}{u_w} = -\frac{1}{2}Re_x^{\frac{1}{2}} f''(0),$$

$$\text{Nu} = \frac{-x\frac{\partial T}{\partial r}(x,R)}{T_w - T_\infty} = -\frac{1}{2}Re_x^{\frac{1}{2}}\, \theta'(x,0),$$

$$\text{Sh} = \frac{-x\frac{\partial C}{\partial r}(x,R)}{C_w - C_\infty} = -\frac{1}{2}Re_x^{\frac{1}{2}}\, C'(x,0),$$

where Re = $\frac{u_w\, x^2}{\nu}$ is the local Reynolds number.

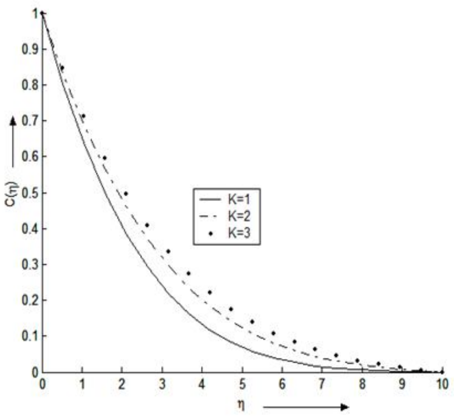

Figure.1. The graph of $C(\eta)$ against η γ =1,ϕ=1, Ha=1, Pr=0.023, λ=1.5, Br=2, S_c=0.6, f_0=0.1 and t_d = 0.001 for various vales of K.

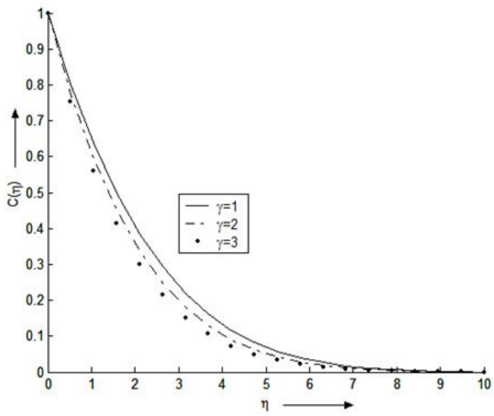

Figure.2. The graph of $C(\eta)$ against η K =1,ϕ=1, Ha=1, Pr=0.023, λ=1.5, Br=2, S_c=0.6, f_0=0.1 and t_d = 0.001 for various vales of γ .

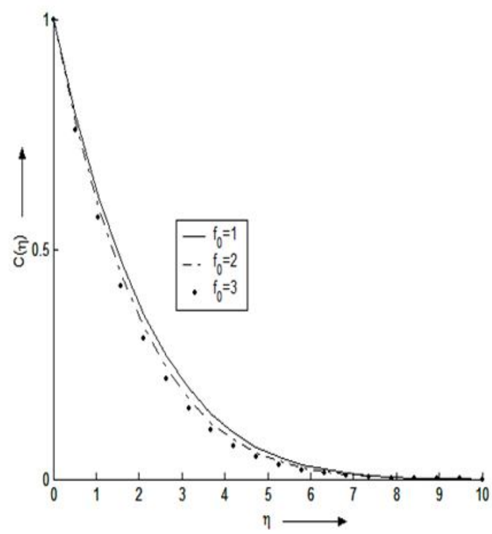

Figure.3. The graph of $C(\eta)$ against η K=1,ϕ=1, Ha=1, Pr=0.023, λ=1.5, Br=2, S_c=0.6, γ =1 and t_d = 0.001 for various vales of f_0.

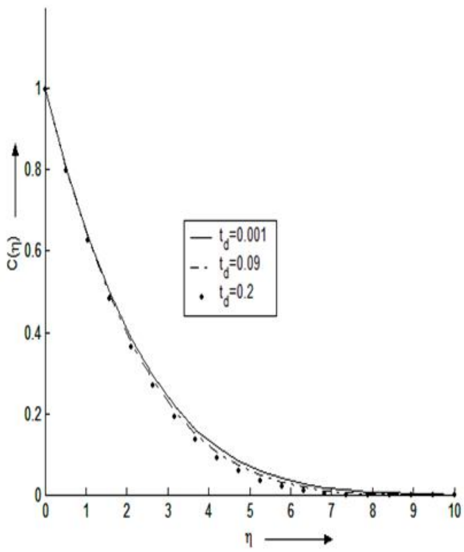

Figure.4. The graph of $C(\eta)$ against η γ =1,ϕ=1, Ha=1, Pr=0.023, λ=1.5, Br=2, S_c=0.6, f_0=0.1 and K= 1 for various vales of t_d.

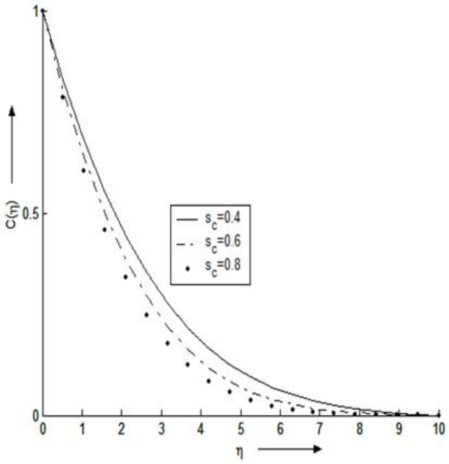

Figure.5. The graph of $C(\eta)$ against η K=1,ϕ=1, Ha=1, Pr=0.023, λ=1.5, Br=2, γ =1, f_0=0.1 and t_d = 0.001 for various vales of S_c.

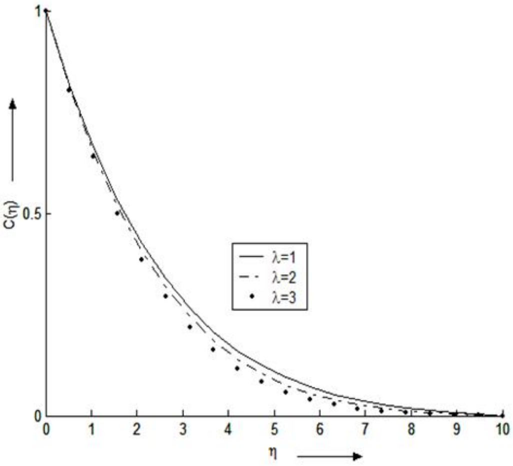

Figure.6. The graph of $C(\eta)$ against η γ =1,ϕ=1, Ha=1, Pr=0.023, K=1, Br=2, S_c=0.6, f_0=0.1 and t_d = 0.001 for various vales of λ.

Finally, the effects of the local skin friction, the Nusselt number and the Sherwood number are tabulated in Table 1. The behaviours of these parameters are self–evident from the Table 1 and hence any further discussion about them seems to be redundant.

Table 1: Numerical values of $f''(0)$, $-\theta'(0)$ and $-C'(0)$ for Pr = 0.023, ϕ=1, Ha=1 and η= 5.

K	γ	f_0	t_d	S_c	λ	$f''(0)$	$-\theta'(0)$	$-C'(0)$
1	1	0.1	0.001	0.6	1.5	0.2741	0.2052	0.4027
2	1	0.1	0.001	0.6	1.5	0.2216	0.2169	0.3401
3	1	0.1	0.001	0.6	1.5	0.1966	0.2213	0.3114
1	1	0.1	0.001	0.6	1.5	0.2741	0.2052	0.4027
1	2	0.1	0.001	0.6	1.5	0.2741	0.2052	0.4697
1	3	0.1	0.001	0.6	1.5	0.2741	0.2052	0.5278
1	1	1	0.001	0.6	1.5	0.2992	0.2065	0.4346
1	1	2	0.001	0.6	1.5	0.3284	0.2080	0.4713
1	1	3	0.001	0.6	1.5	0.3589	0.2095	0.5092
1	1	0.1	0.001	0.6	1.5	0.2741	0.2052	0.4027
1	1	0.1	0.09	0.6	1.5	0.2741	0.2052	0.4083
1	1	0.1	0.2	0.6	1.5	0.2741	0.2052	0.4149
1	1	0.1	0.001	0.4	1.5	0.2741	0.2052	0.3515
1	1	0.1	0.001	0.6	1.5	0.2741	0.2052	0.4027
1	1	0.1	0.001	0.8	1.5	0.2741	0.2052	0.4466
1	1	0.1	0.001	0.6	0.5	-0.1863	0.2028	0.3785
1	1	0.1	0.001	0.6	1	0	0.2039	0.3897
1	1	0.1	0.001	0.6	1.5	0.2741	0.2052	0.4027

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