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Influence of Magnetic Field on Fully Developed Free Convective Flow of a Williamson Fluid through a Porous Medium in a Vertical Channel

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Abstract

In this paper, we investigated the fully developed free convection flow of a Williamson fluid through a porous medium in a vertical channel under the effect of magnetic field. The governing non-linear equations are solved for the velocity field and temperature field using the perturbation technique. The effects of various emerging parameters on the velocity field and temperature field are studied through graphs in detail.

Keywords: MHD; Free Convection; Porous Medium; Williamson Fluid

1. Introduction

In recent years convective heat transfer in porous media has received a great deal of attention due to its importance in various technological applications such as geothermal systems, grain storage, fibre and granular insulation, cooling of electronic systems, packed-sphere beds, chemical catalytic reactors, groundwater hydrology, petroleum reservoirs, coal combustors, nuclear waste repositories and filtration, see

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books by Nakayama [19], Nield and Bejan [20], Ingham and Pop [17,18], Vafai [27] and Pop and Ingham [21].

The investigation of free convection in vertical channels arises in many industrial processes and natural phenomena. Most of the interest in this subject is due to its applications, for instance, in the design of cooling systems for electronic devices and in the field of solar energy collection. Barletta [5] has investigated the combined forced and free flow of a fluid in a vertical channel with viscous dissipation and isothermal-isoflux boundary conditions. Szeri and and Rajagopal [26] have studied the flow of a third grade fluid between heated parallel plates caused by external pressure gradient and obtained similarity solutions of the energy equation, numerically. Akyıldız [1] have studied the flow of third grade fluid between heated parallel plates. Chamka et al. [10] have studied the fully developed free connective flow of micropolar fluid between two vertical parallel plates analytically. Recently, Siddiqui et al. [23] have investigated the flow of a third grade non-Newtonian fluid between two parallel plates separated by a finite gap by using the Adomian decomposition method. Williamson fluid is characterized as a non-Newtonian fluid with shear thinning property, i.e., viscosity decreases with increasing rate of shear stress (Dapra and Scarpi [12]).

The past six decades have seen a tremendous interest in studies involving magnetohydrodynamic flow and heat transfer in porous and non-porous medium. This is primarily due to an increase in industrial and technological applications of flows involving electrically conduction fluids. For example, Sparrow and Cess [24] considered the effect of a magnetic field on the free convection heat transfer from a surface. Garandet et al. [14] have studied buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. Chamkha [9] have investigated the free convection effects on three-dimensional flow over a vertical stretching surface in the presence of a magnetic field. Bhargava et al. [3] have studied the effect of magnetic field on the free convection flow of micropolar fluid between two parallel porous vertical plates. Hayat et al. [15] have studied the Hall effects on the unsteady hydromagnetic oscillatory flow of a second grade fluid. Hazeem attia [16] have investigated the unsteady flow of a dusty conducting fluid between parallel porous plates. Sanyal and Adhikari [22] have studied the effects of radiation on MHD fluid flow in vertical channel. Recently, Subramanyam et al. [25] have investigated the fully developed free convection flow of a third grade fluid through a porous medium in a vertical channel under the effect of a magnetic field.

In view of these, we studied the fully developed free convection flow of a Williamson fluid through a porous medium in a vertical channel under the effect of magnetic field. The governing non-linear equations are solved for the velocity field and temperature field using the perturbation technique. The effects of various emerging parameters on the velocity field and temperature field are studied through graphs in detail.

2. Mathematical formulation

The equations governing the flow of an incompressible Williamson fluid are given by

$$\nabla . V = 0 \tag{1}$$

$$\rho \frac{dV}{dt} = \rho f + \nabla . \tau \tag{2}$$

where ρ denotes the constant fluid density, V is the velocity vector and f represents the body force per unit mass. The operator d/dt denotes the material time derivative and τ is the stress tensor.

The constitutive equation for a Williamson fluid is given by

$$\tau = -\left[\eta_\infty + (\eta_0 + \eta_\infty)(1 - \Gamma\dot{\gamma})^{-1}\right]\dot{\gamma} \tag{3}$$

Where τ is the extra stress tensor, η_∞ is the infinite shear rate, viscosity η_o is the zero shear rate viscosity, Γ is the time constant and $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\pi} \tag{4}$$

where π is the second invariant stress tensor. We consider in the constitutive Eq. (3) the case for which $\eta_\infty = 0$ and $\Gamma\dot{\gamma} < 1$ so we can write.

$$\tau = -\eta_0 (1 + \Gamma\dot{\gamma})\dot{\gamma} \tag{5}$$

The above model reduces to Newtonian for $\Gamma = 0$.

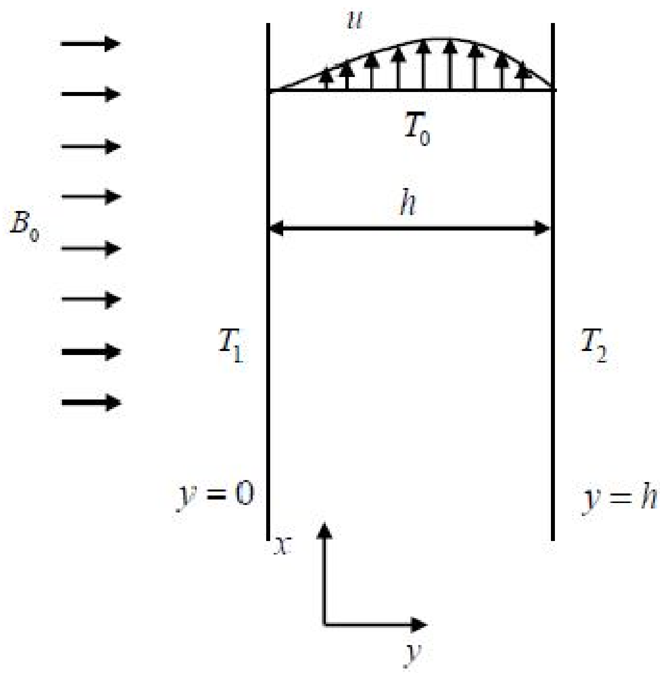

Fig. 1 The physical model

We consider the laminar free convection flow of a Williamson fluid between two plates at distance h a apart filled with porous medium, as shown in Fig.1.We choose co-ordinates system, with X - axis parallel to the flow while Y - axis is normal to the flow. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The flow assume steady and fully developed, i.e., the transverse velocity is zero. It is also assumed that the walls are heated uniformly but their temperatures may be different resulting in asymmetric heating situation under these assumptions the equations that describe the physical situation are

$$\eta_0 \frac{d^2u}{dy^2} + \Gamma \frac{d}{dy}\left[\left(\frac{du}{dy}\right)^2\right] - \frac{\eta_0}{k}u - \sigma B_0^2 u + \rho g \beta (T - T_0) = 0 \tag{6}$$

$$\frac{\partial^2 T}{\partial y^2} = 0 \tag{7}$$

where k is permeability of the porous medium and σ is the electrical conductivity.

Subject to the boundary conditions

$$u(0) = 0, \quad T(0) = T_1, \; u(h) = 0, \quad T(h) = T_2 \tag{8}$$

Introducing the following non-dimensional variables

$$\bar{u} = \frac{u}{U}, \bar{y} = \frac{y}{h}, \bar{x} = \frac{x}{h}, We = \frac{U\Gamma}{\eta_0 h}, \theta = \frac{T - T_0}{T_2 - T_0}, r_T = \frac{T_1 - T_0}{T_2 - T_0} \tag{9}$$

into Eqs. (6) and (7), we get (after dropping the bars)

$$\frac{d^2u}{dy^2} + We\frac{d}{dy}\left[\left(\frac{du}{dy}\right)^2\right] - N^2 u + \frac{Gr}{\mathrm{Re}}\theta = 0 \tag{10}$$

$$\frac{d^2\theta}{dy^2} = 0 \tag{11}$$

where $N = \sqrt{M^2 + \frac{1}{Da}}$, $M = B_0 h\sqrt{\frac{\sigma}{\eta_0}}$ is the Hartmann number, $Da = \frac{k}{h^2}$ is the Darcy number, $Gr = \frac{g\beta(T_2 - T_0)h^3}{\nu^2}$ is the Grashof number and $\mathrm{Re} = \frac{Uh}{\nu}$ is the Reynolds number.

The corresponding dimensionless boundary conditions

$$u(0) = 0, \quad \theta(0) = r_T, \; u(1) = 0, \quad \theta(1) = 1 \tag{12}$$

3. Perturbation Solution

Eq. (10) is non-linear and it is difficult to get a closed form solution. However for vanishing We, the boundary value problem is agreeable to an easy analytical solution. In this case the equation becomes linear and can be solved. Nevertheless, small Γ suggests the use of perturbation technique to solve the non-linear problem. Accordingly, we write

$$u = u_0 + Weu_1 \tag{13}$$

and

$$\theta = \theta_0 + We\theta_1 \tag{14}$$

Substituting equations (11) and (12) into Eqs. (8) and (9) and boundary conditions (10) and then equating the like powers of We, we obtain

3.1 Zeroth-order system $\left(We^0\right)$

$$\frac{d^2u_0}{dy^2} - N^2u_0 = -\frac{Gr}{\mathrm{Re}}\theta_0 \tag{15}$$

$$\frac{d^2\theta_0}{dy^2} = 0 \tag{16}$$

Together with boundary conditions

$$u_0(0) = u_0(1) = 0,\ \theta_0(0) = r_T,\ \theta_0(1) = 1 \tag{17}$$

3.2 First-order system (We)

$$\frac{d^2u_1}{dy^2} - N^2u_1 = -\frac{d}{dy}\left[\left(\frac{du_0}{dy}\right)^2\right] - \frac{Gr}{\mathrm{Re}}\theta_1 \tag{18}$$

$$\frac{d^2\theta_1}{dy^2} = 0 \tag{19}$$

Together with boundary conditions

$$u_1(0) = u_1(1) = 0,\ \theta_1(0) = 0,\ \theta_1(1) = 0 \tag{20}$$

3.3 Zeroth-order solution

Solving Eqs. (15) and (16) using the boundary conditions (20), we get

$$\theta_0 = r_T + (1 - r_T)y \tag{21}$$

$$u_0 = \frac{Gr}{\mathrm{Re}}\frac{1}{N^2}\left[C_1 \sinh Ny - r_T \cosh Ny + (1 - r_T)y + r_T\right] \tag{22}$$

here

$$C_1 = \frac{r_T \cosh N - 1}{\sinh N}.$$

3.4 First-order solution

Solving Eq. (19) subject to the boundary conditions in Eq. (20), we get

$$\theta_1 = 0 \tag{23}$$

Substituting the Eqs. (22) and (23) into the Eq. (18) and then solving the resulting equation with the corresponding conditions, we get

$$u_1 = \left(\frac{Gr}{\mathrm{Re}}\right)^2 \frac{1}{6N^6}\begin{bmatrix} -2C_3 \cosh Ny + 6C_7 N^2 \sinh Ny - 2C_2 \sinh 2Ny \\ +2C_3 \cosh 2Ny + 3C_4 Ny \sinh Ny - 3C_5 Ny \cosh Ny \end{bmatrix} \tag{24}$$

where $C_2 = N^3\left(r_T^2 + C_1^2\right)$, $C_3 = 2A_1 r_T N^3$, $C_4 = 2r_T\left(1 - r_T\right)N^2$, $C_5 = 2C_1\left(1 - r_T\right)N^2$,

$$C_6 = \frac{1}{6N^2}\left[2C_2 \sinh 2N - 2C_3 \cosh 2N - 3NC_4 \sinh N + 3NC_5 \cosh N\right],$$

$$C_7 = \left[C_6 + \frac{C_3 \cosh N}{3N^2}\right] \Big/ \sinh N.$$

Finally, the perturbation solutions up to first order for θ and u are given by

$$\theta = \theta_0 + \Gamma\theta_1 = \theta_0 = r_T + \left(1 - r_T\right)y \tag{25}$$

and

$$u = u_0 + \Gamma u_1 \tag{26}$$

4. Discussion of the results

The effect of Weissenberg number We on u for $M = 1$, $r_T = 0.5$, $Gr = 1$ and $\mathrm{Re} = 1$ is shown in Fig. 2. It is observed that, velocity u first decreases and then increases with increasing We.

Fig. 3 shows the effect of Darcy number Da on u for $Gr = 1$, $We = 0.2$, $M = 1$, $r_T = 0.5$ and $\mathrm{Re} = 1$. It is noted that, the velocity u increases with increasing Da.

The effect of Hartman number M on u for $We = 0.1$, $r_T = 0.5$, $Gr = 1$ and $\mathrm{Re} = 1$ is represented in Fig. 4. It is found that, the velocity u decreases with an increase in Hartmann number M.

Fig. 5 depicts the effect of Grashof number Gr on u for $M = 1, r_T = 0.5$, $We = 0.1$ and $\mathrm{Re} = 1$. It is observed that, the velocity u increases with increasing Grashof number Gr.

The effect of Reynolds number Re on u for $M=1, r_T=0.5, Gr=1$ and $We=0.1$ is shown in Fig. 6. It is noted that, the velocity u decreases with an increase in Reynolds number Re.

Fig. 7 illustrates the effect of wall temperature parameter r_T on u for $M=1$, $We=0.1$, $Gr=1$ and $\mathrm{Re}=1$. It is found that, the velocity u increases with increasing r_T.

The effect of wall temperature parameter r_T on θ is shown in Fig. 8. It is observed that, the temperature θ increases with an increase in r_T.

5. Conclusions

In this paper, we studied the fully developed free convection flow of a Williamson fluid in a vertical channel under the effect of magnetic field. The governing non-linear equations are solved for the velocity field and temperature field using the perturbation technique. It is found that, the velocity increases with increasing We, Gr and r_T, while it decreases with increasing M. It is observed that, the temperature θ increases with an increase in r_T.

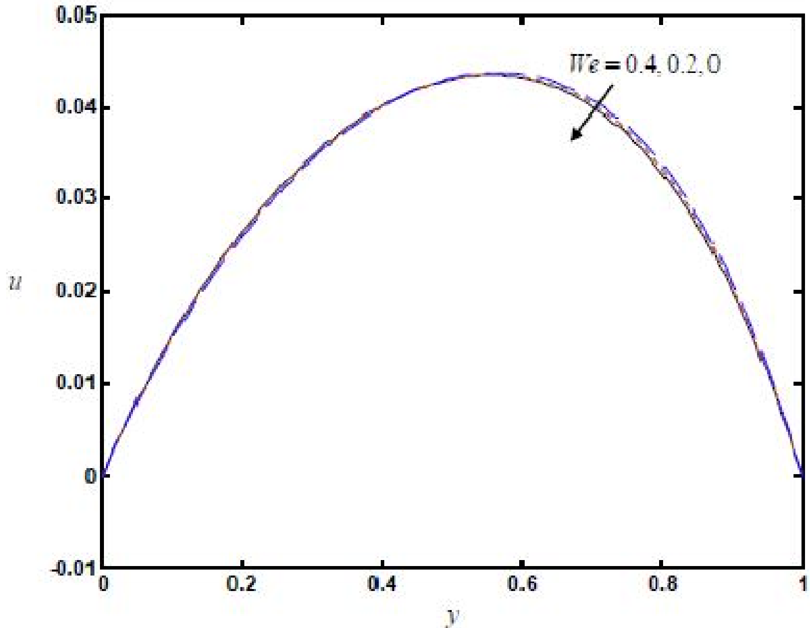

Fig. 2. Effect of Weissenberg number We on u for $Gr=1$, $Da=0.1$, $M=1$, $r_T=0.5$ and $\mathrm{Re}=1$.

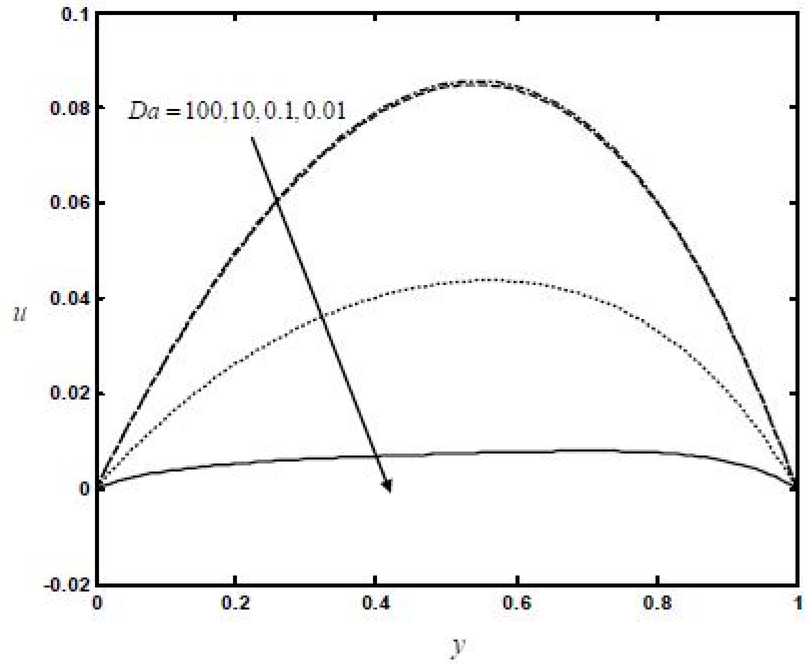

Fig. 3. Effect of Darcy number Da on u for $Gr = 1$, $We = 0.2$, $M = 1$, $r_T = 0.5$ and $\mathrm{Re} = 1$.

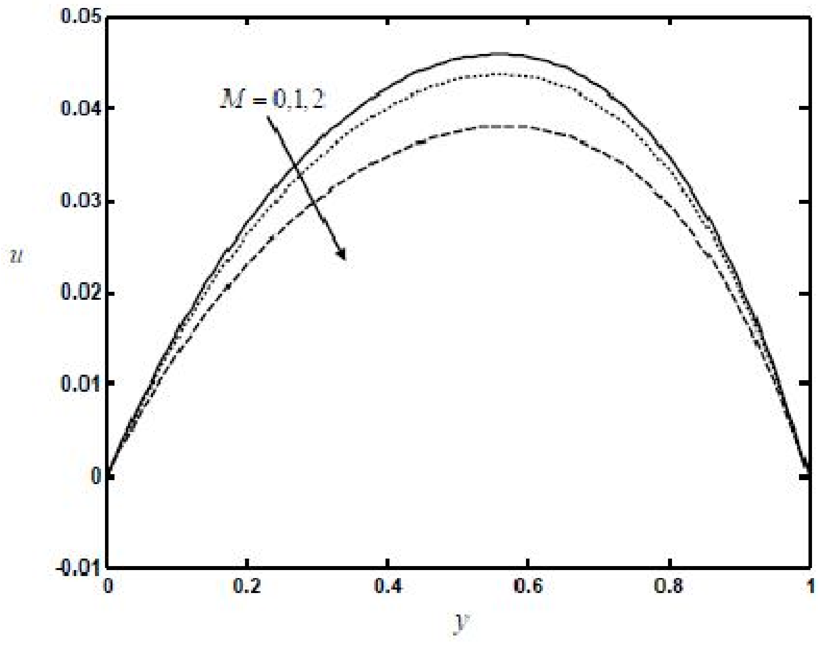

Fig. 4. Effect of Hartmann number M on u for $We = 0.2, Da = 0.1, r_T = 0.5,$ $Gr = 1$ and $\mathrm{Re} = 1$.

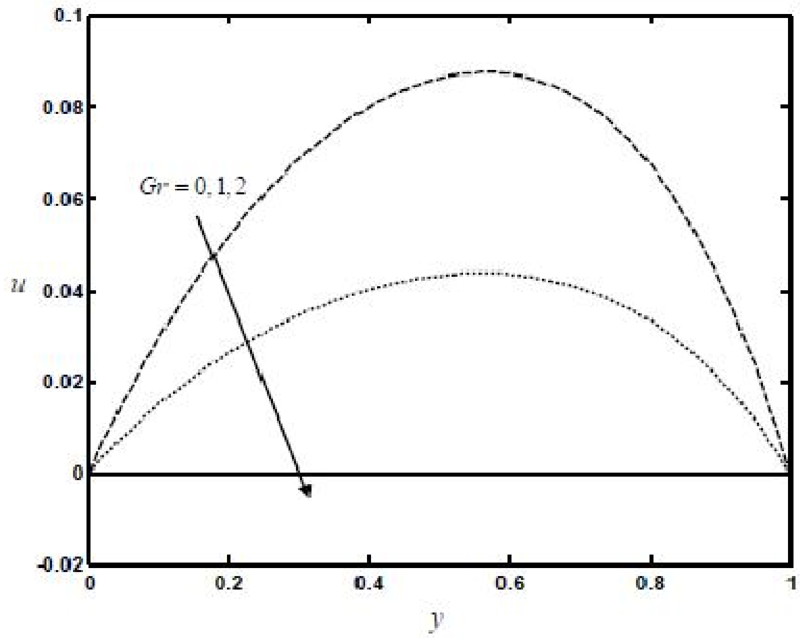

Fig. 5. Effect of Grashof number Gr on u for $M=1, Da=0.1, r_T=0.5,$ $We=0.1$ and $\mathrm{Re}=1$.

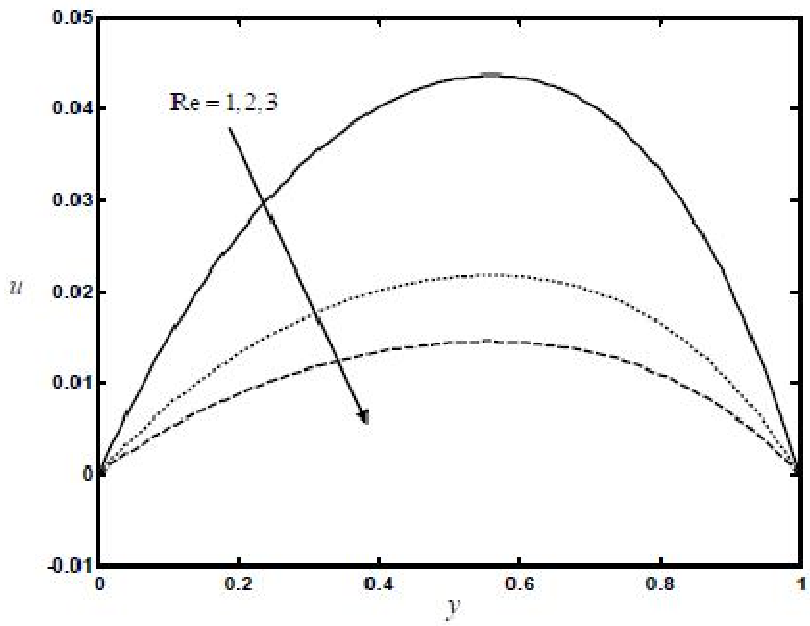

Fig. 6. Effect of Reynolds number Re on u for $M=1, r_T=0.5,$ $We=0.1$ and $Gr=1$.

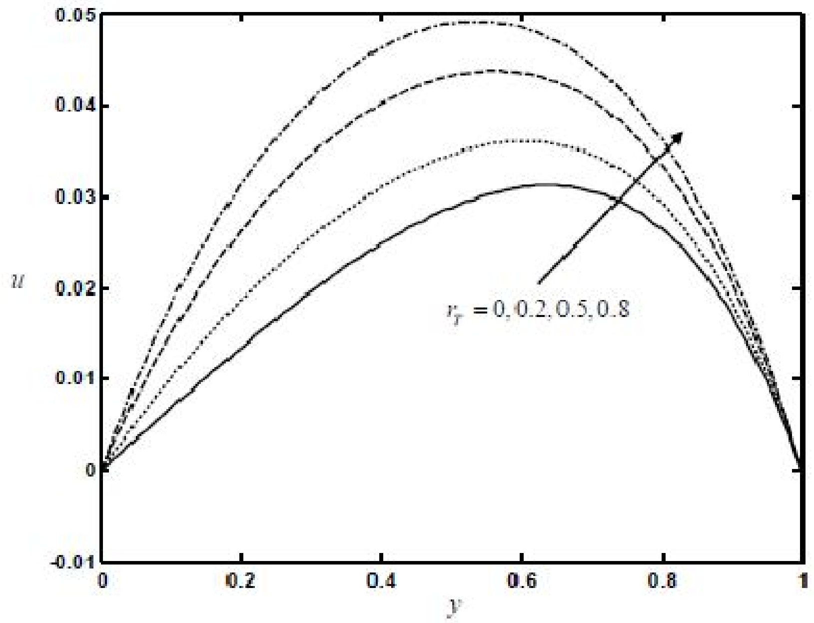

Fig. 7. Effect of wall temperature parameter r_T on u for $M=1$, $Gr=1$, $We=0.1$ and $\mathrm{Re}=1$.

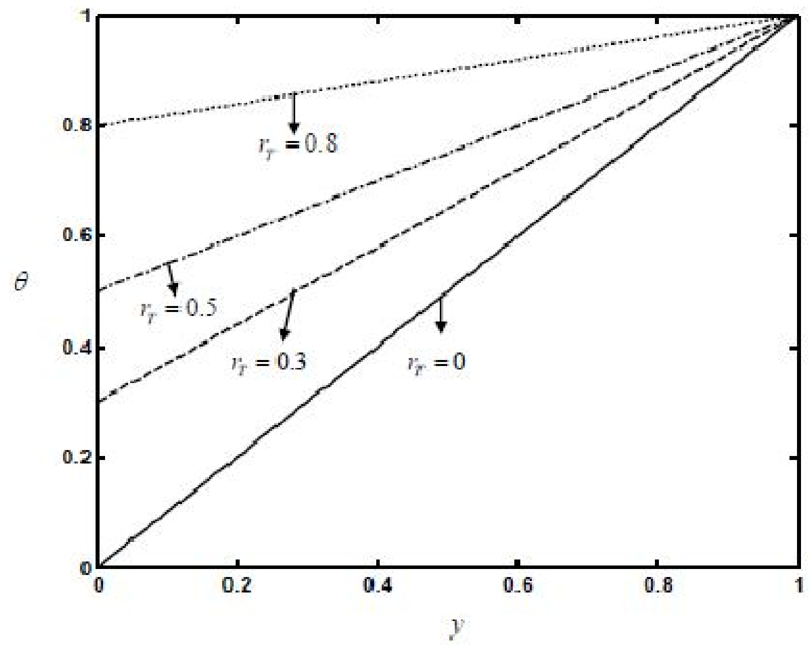

Fig. 8. Effect of wall temperature parameter r_T on θ .

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