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Radiation, Soret and Dufour Effects in MHD Channel Flow Bounded by a Long Wavy Wall and a Uniformly Moving Parallel Flat Wall

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ABSTRACT:

A parametric study to investigate the effects of Radiation, Soret and Dufour effects on a two dimensional free convective MHD flow of a viscous incompressible and electrically conducting fluid through a channel bounded by a long vertical wavy wall and a uniformly moving parallel flat wall is presented. A uniform magnetic field is assumed to be applied normal to the flat wall. The equations governing fluid flow are solved analytically subject to the relevant boundary conditions. It is assumed that the solution consists of two parts, a mean part and a perturbed part. The long wave approximation has been used to obtain the solution of the perturbed part and to solve the mean part the well known approximation used by Ostrich (1952) [1] has been utilized. The perturbed part of the solution is the contribution from the waviness of the wall. The expressions for zeroth and first order velocity, temperature, concentration, and skin friction and the rates of heat and mass transfer at the walls are obtained. Some of the results indicating the influence of radiation, Soret and Dufour effects on the above fields have been presented graphically.

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KEYWORDS: MHD, Free convection, Radiation, Thermal diffusion, Diffusion-thermo

INTRODUCTION:

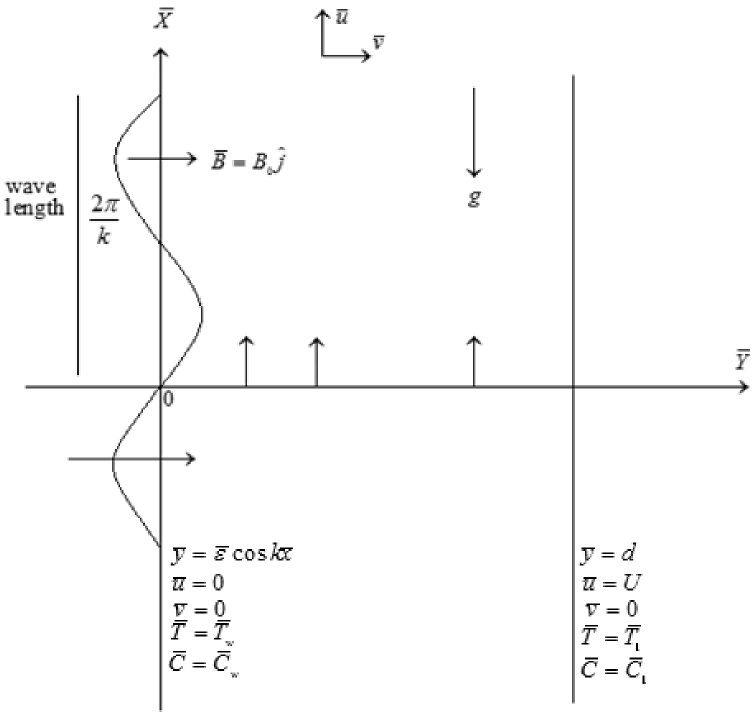
The incompressible boundary layer flow over a wavy wall has drawn attention due to its application in several areas such as cross-hatching on ablative surface, transpiration cooling of re-entry vehicle and rocket booster and film vaporization in combustion chambers. Lukodius, Nayfeh and Saric [2] made a linear analysis of compressible boundary layer flows over a wavy wall. The Rayleigh problem for wavy wall was studied by Shankar and Sinha [3]. The analysis of the effect of small amplitude wall waviness upon the stability of the laminar boundary layer was made by Lessen and Gangwani [4]. Vajravelu and Sastri [5] presented an analysis of the free convective heat transfer in a viscous incompressible fluid between a long vertical wavy wall and a parallel flat wall. Further they extended their work for vertical wavy channels. Rao and Sastri [6] extended the work of Vajravelu and Sastri [7] to viscous heating effects when the fluid properties are constant. Again Rao [8] reinvestigated the problem of Rao and Sastri [6] for the channels which are of different wave numbers. Das and Ahmed [9] studied the free convection MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. In the above mentioned works, the diffusion-thermo (Dufour) and the thermal-diffusion (Soret) terms were not taken into account in the energy and concentration equations respectively. But when the heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and driving potentials are of a more intricate nature. It is found that a heat flux can be generated not only by temperature gradients but by composition gradients as well. The heat flux that occurs due to composition gradient is called the Dufour effect or diffusion-thermo effect. On the other hand the flux of mass caused due to temperature gradient is known as the Soret effect or the thermal-diffusion effect. The experimental investigation of the thermal-diffusion effect on mass transfer related problems was first done by Charles Soret in 1879. Hence this thermal-diffusion is known as the Soret effect in honour of Charles Soret. In general the Soret and Dufour effects are of a smaller order of magnitude than the effects described in Fourier’s or Fick’s law and are often neglected in heat and mass transfer processes. Though these effects are quite small, certain devices can be arranged to produce very steep temperature and concentration gradients so that the separation of components in mixtures are affected. Eckert and Drake [10] have emphasized that the Soret effect assumes significance in cases concerning isotope separation and in mixtures between gases with very light molecular weight (H_2, H_e) and for medium molecular weight (N_2, air), the Dufour effect is found to be of considerable magnitude such that it cannot be ignored. Following Eckert and Drake’s work [10] several other investigators have carried out model studies on the Soret and Dufour effect in different heat and mass transfer problems. Some of them are Durnskaya and Worek [11], Kafoussias and Williams [12], Sattar and Alam [13], Alam et al. [14] and Raju et al. [15]. Recently Ahmed et al. [16] have investigated the Soret and Dufour effects in free convection MHD flow of a viscous incompressible fluid through a channel bounded by a long vertical wavy wall and parallel flat wall.

Radiation is a process of heat transfer through electromagnetic waves. Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, and solar power technology and space vehicle re-entry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role in space related technology. The effect of radiation on various convective flows under different conditions has been studied by many researchers including Hossain and Takhar [17], Ahmed and Sarmah [18], Rajesh and Varma [19] and Kesavaiah et al. [20].

The present authors are aware that no attempt has been made till now, to study the effect of thermal radiation on an MHD free convective mass transfer flow through a channel bounded by a long wavy wall and a uniformly moving parallel flat wall involving Soret and Dufour effects. Such an attempt has been made in the present paper in view of the application of such types of problems in different engineering fields. This work is a generalization of the work done by Ahmed et al. [16], to incorporate the radiation effect in addition to Soret and Dufour effects. In particular, the present work is an extension to that of Ahmed and Bhattacharyya [22], to the case of a moving flat wall and a fixed wavy wall.

BASIC EQUATIONS:

We consider a two dimensional steady laminar free convective MHD flow through a vertical channel. The x-axis is taken parallel to the flat wall and y-axis is perpendicular to it. The wavy and the flat walls are represented by $\bar{y} = \bar{\varepsilon} \cos k\bar{x}$ and $\bar{y} = d$ respectively, $\bar{T}_w$ and $\bar{T}_1$ being their constant temperatures.



Flow Configuration

Our investigation is restricted to the following assumptions:

1. All the fluid properties, except the density in the buoyancy force term, are constants.
2. The viscous and magnetic dissipation of energy are negligible.
3. The volumetric heat source/sink term in the energy equation is constant.
4. The magnetic Reynolds number is small enough to neglect the induced magnetic field.
5. The wave length of the wavy wall, which is proportional to $1/k$, is large.

Under the foregoing assumptions, the equations which govern the two dimensional steady laminar free convective MHD flow and heat transfer in a viscous incompressible fluid occupying the channel are as follows:
The momentum equations:

$$\rho\left[\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right]=-\frac{\partial\bar{p}}{\partial\bar{x}}+\mu\left[\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right]-\rho g-\sigma\bar{B}^2\bar{u} \tag{1}$$

$$\rho\left[\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right]=-\frac{\partial\bar{p}}{\partial\bar{y}}+\mu\left[\frac{\partial^2\bar{v}}{\partial\bar{x}^2}+\frac{\partial^2\bar{v}}{\partial\bar{y}^2}\right] \tag{2}$$

The continuity equation:

$$\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}=0 \tag{3}$$

The energy equation:

$$\rho C_p\left[\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}\right]=\kappa\left[\frac{\partial^2\bar{T}}{\partial\bar{x}^2}+\frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right]+\frac{\rho D_M K_T}{C_S}\left[\frac{\partial^2\bar{C}}{\partial\bar{x}^2}+\frac{\partial^2\bar{C}}{\partial\bar{y}^2}\right]+Q-\frac{\partial\bar{q}_r}{\partial\bar{y}} \tag{4}$$

The species continuity equation:

$$\bar{u}\frac{\partial\bar{C}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{C}}{\partial\bar{y}}=D_M\left[\frac{\partial^2\bar{C}}{\partial\bar{x}^2}+\frac{\partial^2\bar{C}}{\partial\bar{y}^2}\right]+\frac{D_M K_T}{T_m}\left[\frac{\partial^2\bar{T}}{\partial\bar{x}^2}+\frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right] \tag{5}$$

The radiative heat flux $\bar{q}_r$ as emphasised by Cogely et al. [21] for an optically thin fluid is given by:

$$\frac{\partial\bar{q}_r}{\partial\bar{y}}=4I\left(\bar{T}-\bar{T}_S\right)$$

$$\text{where, } I=\int_0^\infty (K_\lambda)_w\left(\frac{\partial e_{b\lambda}}{\partial\bar{T}}\right)_w d\lambda \tag{6}$$

In static condition (1) takes the form

$$0=-\frac{\partial\bar{p}_s}{\partial\bar{x}}-\rho_s g \tag{7}$$

Now, (1) and (7) yields:

$$\rho\left[\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right]=-\frac{\partial}{\partial\bar{x}}\left(\bar{p}-\bar{p}_s\right)+\mathrm{g}\left(\rho_s-\rho\right)+\mu\left[\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right]-\sigma B^2\bar{u} \tag{8}$$

The equation of state is given by:

$$\rho = \rho_s \left[1 - \beta \left(\bar{T} - \bar{T}_s \right) - \bar{\beta} \left(\bar{C} - \bar{C}_s \right) \right] \tag{9}$$

The equation (8) and (9) together give:

$$\rho \left[\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right] = -\frac{\partial}{\partial \bar{x}} \left(\bar{p} - \bar{p}_s \right) + \rho g \left[\beta \left(\bar{T} - \bar{T}_s \right) + \bar{\beta} \left(\bar{C} - \bar{C}_s \right) \right] + \mu \left[\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right] - \sigma B^2 \bar{u} \tag{10}$$

The relevant boundary conditions are as under:

$$\bar{y} = \bar{\varepsilon} \cos k\bar{x} : \quad \bar{u} = \bar{v} = 0; \quad \bar{T} = \bar{T}_w; \quad \bar{C} = \bar{C}_w \tag{11}$$

$$\bar{y} = d : \quad \bar{u} = \bar{U}; \quad \bar{v} = 0; \quad \bar{T} = \bar{T}_1; \quad \bar{C} = \bar{C}_1 \tag{12}$$

We define the following non-dimensional quantities:

$$x = \frac{\bar{x}}{d},\ y = \frac{\bar{y}}{d},\ u = \frac{\bar{u}d}{\upsilon},\ v = \frac{\bar{v}d}{\upsilon},\ p = \frac{\bar{p}d^2}{\rho\upsilon^2},\ p_s = \frac{\bar{p}_s d^2}{\rho\upsilon^2},\ \lambda = kd,\ \varepsilon = \frac{\bar{\varepsilon}}{d},\ p_r = \frac{\mu C_p}{k},$$

$$G_r = \frac{d^3 g \beta \left(\bar{T}_w - \bar{T}_s \right)}{\upsilon^2},\ G_m = \frac{d^3 g \bar{\beta} \left(\bar{C}_w - \bar{C}_s \right)}{\upsilon^2},\ n = \frac{\bar{C}_1 - \bar{C}_s}{\bar{C}_w - \bar{C}_s},\ m = \frac{\bar{T}_1 - \bar{T}_s}{\bar{T}_w - \bar{T}_s},$$

$$S_r = \frac{D_M K_T \left(\bar{T}_w - \bar{T}_s \right)}{\upsilon T_m \left(\bar{C}_w - \bar{C}_s \right)},\ D_u = \frac{D_M K_T \left(\bar{C}_w - \bar{C}_s \right)}{\upsilon C_s C_p \left(\bar{T}_w - \bar{T}_s \right)},\ S_c = \frac{\upsilon}{D_M},\ M = \frac{\sigma B^2 d^2}{\rho \upsilon},$$

$$T = \frac{\bar{T} - \bar{T}_s}{\bar{T}_w - \bar{T}_s},\ C = \frac{\bar{C} - \bar{C}_s}{\bar{C}_w - \bar{C}_s},\ \alpha = \frac{Qd^2}{k \left(\bar{T}_w - \bar{T}_s \right)},\ N = \frac{4Id^2}{\rho \upsilon C_p},\ U = \frac{\bar{U}d}{\upsilon}$$

All physical variables and parameters are defined in the Nomenclature section. The corresponding equations in non dimensional form are as under:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial}{\partial x} \left(p - p_s \right) + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + G_r T + G_m C - Mu \tag{13}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \tag{14}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + P_r D_u \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) = P_r \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) - \alpha + P_r N T \tag{15}$$

$$\frac{1}{S_c} \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + S_r \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \tag{16}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{17}$$

Subject to the folowing boundary conditions:

$$u = 0,\ v = 0,\ T = 1,\ C = 1 \quad \text{on} \quad y = \varepsilon \cos \lambda x \tag{18}$$

$$u = U,\ v = 0,\ T = m,\ C = n \quad \text{on} \quad y = 1 \tag{19}$$

METHOD OF SOLUTION:

In order to solve the equations (13) to (17), we assume $u,\ v,\ p,\ T$ and C as follows:

$$u(x,y)=u_0(y)+\varepsilon u_1(x,y)+ \text{-----------} \tag{20.1}$$

$$v(x,y)=\varepsilon v_1(x,y)+ \text{---------------} \tag{20.2}$$

$$p(x,y)=p_0(x)+\varepsilon p_1(x,y)+ \text{-----------} \tag{20.3}$$

$$T(x,y)=T_0(y)+\varepsilon T_1(x,y)+ \text{-----------} \tag{20.4}$$

$$C(x,y)=C_0(y)+\varepsilon C_1(x,y)+ \text{-----------} \tag{20.5}$$

By substituting the transformations (20.1) to (20.5) in (13) to (17), and by equating the coefficients of ε^0, ε^1 and neglecting the higher powers of ε and assuming $\frac{\partial}{\partial x}(p_0 - p_s) = 0,$ (following Ostrach [1]) we derive the following set of ordinary differential equations:

$$\frac{d^2u_0}{dy^2} - Mu_0 = -G_rT_0 - G_mC_0 \tag{21}$$

$$\frac{d^2T_0}{dy^2} + P_rD_u\frac{d^2C_0}{dy^2} = -\alpha + P_rNT_0 \tag{22}$$

$$\frac{d^2C_0}{dy^2} + S_rS_c\frac{d^2T_0}{dy^2} = 0 \tag{23}$$

$$u_0\frac{\partial u_1}{\partial x} + v_1\frac{du_0}{dy} = -\frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + G_rT_1 + G_mC_1 - Mu_1 \tag{24}$$

$$u_0\frac{\partial v_1}{\partial x} = -\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} \tag{25}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \tag{26}$$

$$\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} + P_rD_u\left(\frac{\partial^2 C_1}{\partial x^2} + \frac{\partial^2 C_1}{\partial y^2}\right) = P_r\left(u_0\frac{\partial T_1}{\partial x} + v_1\frac{dT_0}{dy}\right) + P_rNT_1 \tag{27}$$

$$\frac{1}{S_c}\left(\frac{\partial^2 C_1}{\partial x^2} + \frac{\partial^2 C_1}{\partial y^2}\right) + S_r\left(\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2}\right) = u_0\frac{\partial C_1}{\partial x} + v_1\frac{dC_0}{dy} \tag{28}$$

Subject to the following boundary conditions:

$$\left.\begin{array}{l} u_0 = 0,\ T_0 = 1,\ \ C_0 = 1 \quad \text{at} \ \ y = 0 \\ u_0 = U,\ T_0 = m,\ \ C_0 = n \quad \text{at} \ \ y = 1 \end{array}\right\} \tag{29}$$

$$\left.\begin{array}{l} u_1 = -\mathrm{Re}\left[u_0'(0)e^{i\lambda x}\right],\ v_1 = 0,\ T_1 = -\mathrm{Re}\left[T_0'(0)e^{i\lambda x}\right],\ C_1 = -\mathrm{Re}\left[C_0'(0)e^{i\lambda x}\right] \ \text{at} \ y = 0 \\ u_1 = 0,\ v_1 = 0,\ T_1 = 0,\ C_1 = 0 \ \text{at} \ y = 1 \end{array}\right\} \tag{30}$$

The solutions of the equations (21), (22) and (23) subject to the boundary conditions (29) are:

$$u_0(y) = A_{20}e^{\sqrt{M}y} + A_{19}e^{-\sqrt{M}y} + A_{15}e^{A_1y} + A_{16}e^{A_2y} + A_{18}y + A_{17} \tag{31}$$

$$T_0(y) = A_4e^{A_1y} + A_3e^{A_2y} + A_5 \tag{32}$$

$$C_0(y) = A_8 e^{A_1 y} + A_9 e^{A_2 y} + A_7 y + A_{10} \tag{33}$$

Where,

$$A_1 = \sqrt{\frac{P_r N}{1 - S_c S_r P_r D_u}}\ ,\quad A_2 = -A_1\ ,\quad A_3 = \frac{1}{e^{A_2} - e^{A_1}}\left(m - \frac{\alpha}{P_r N}\right) - \frac{e^{A_1}}{e^{A_2} - e^{A_1}}\left(1 - \frac{\alpha}{P_r N}\right),$$

$$A_4 = 1 - \frac{\alpha}{P_r N} - A_3\ ,\qquad A_5 = \frac{\alpha}{P_r N}\ ,\qquad A_6 = n + S_c S_r\left(A_4 e^{A_1} + A_3 e^{A_2}\right),$$

$$A_{10} = 1 + S_c S_r\left(A_4 + A_3\right),\qquad A_7 = A_6 - A_{10}\ ,\qquad A_8 = -S_c S_r A_4\ ,\qquad A_9 = -S_c S_r A_3\ ,$$

$$A_{11} = -\left(G_r A_4 + G_m A_8\right),\qquad A_{12} = -\left(G_r A_3 + G_m A_9\right),\qquad A_{13} = -G_m A_7,$$

$$A_{14} = -\left(G_r A_5 + G_m A_{10}\right),\qquad A_{15} = \frac{A_{11}}{A_1^2 - M},\qquad A_{16} = \frac{A_{12}}{A_2^2 - M},\qquad A_{17} = -A_{14}/M\ ,$$

$$A_{18} = -A_{13}/M\ ,$$

$$A_{19} = -\frac{1}{e^{\sqrt{M}} - e^{-\sqrt{M}}}\left[\left(A_{15} + A_{16} + A_{17}\right)e^{\sqrt{M}} - A_{15}e^{A_1} - A_{16}e^{A_2} - A_{17} - A_{18} + U\right],$$

$$A_{20} = -A_{19} - A_{15} - A_{16} - A_{17}$$

Now in order to obtain the solution of the first order equations, we introduce the stream function $\bar{\psi}_1$ defined by:

$$u_1 = -\frac{\partial \bar{\psi}_1}{\partial y}\ ,\qquad v_1 = \frac{\partial \bar{\psi}_1}{\partial x} \tag{34}$$

On elimination of p , the equations (24), (25), (27) and (28) yield:

$$u_0\left(\bar{\psi}_{1,xyy} + \bar{\psi}_{1,xxx}\right) - u_0''\bar{\psi}_{1,x} = \bar{\psi}_{1,xxxx} + 2\bar{\psi}_{1,xxyy} + \bar{\psi}_{1,yyyy} - G_r T_{1,y} - G_m C_{1,y} - M\bar{\psi}_{1,yy} \tag{35}$$

$$T_{1,xx} + T_{1,yy} + P_r D_u\left(C_{1,xx} + C_{1,yy}\right) = P_r\left(u_0 T_{1,x} + \bar{\psi}_{1,x} T_0'\right) + P_r N T_1 \tag{36}$$

$$\frac{1}{S_c}\left(C_{1,xx} + C_{1,yy}\right) + S_r\left(T_{1,xx} + T_{1,yy}\right) = u_0 C_{1,x} + \bar{\psi}_{1,x} C_0' \tag{37}$$

Considering the transformations $\bar{\psi}_1 = e^{i\lambda x}\psi(y)$, $T_1 = e^{i\lambda x}\theta(y)$, $C_1 = e^{i\lambda x}\phi(y)$ the equations (35), (36) and (37) reduce to

$$\psi^{iv} - \psi''\left(i\lambda u_0 + 2\lambda^2 + M\right) + \psi\left(i\lambda u_0'' + i\lambda^3 u_0 + \lambda^4\right) = G_r\theta' + G_m\phi' \tag{38}$$

$$\theta'' - \theta'\left(\lambda^2 + P_r N + P_r u_0 i\lambda\right) + P_r D_u\left(-\lambda^2\phi + \phi''\right) = P_r i\lambda\psi T_0' \tag{39}$$

$$\phi'' - \phi\left(\lambda^2 + S_c u_0 i\lambda\right) + S_c S_r\left(-\lambda^2\theta + \theta''\right) = i\lambda S_c \psi C_0' \tag{40}$$

Subject to the relevant boundary conditions:

$$\psi'(y) = u_0'(0),\quad \psi(y) = 0,\quad \theta(y) = -T_0'(0),\quad \phi(y) = -C_0'(0)\quad \text{at } y = 0 \tag{41}$$

$$\psi'(y) = 0,\quad \psi(y) = 0,\quad \theta(y) = 0,\quad \phi(y) = 0\quad \text{at } y = 1 \tag{42}$$

We assume the series expansion for ψ, θ and ϕ as follows:

$$\psi = \psi_0(y) + \lambda\psi_1(y) + \lambda^2\psi_2(y) + ----------- \tag{43}$$

$$\theta = \theta_0(y) + \lambda\theta_1(y) + \lambda^2\theta_2(y) + ----------- \tag{44}$$

$$\phi=\phi_0(y)+\lambda\phi_1(y)+\lambda^2\phi_2(y)+------------ \tag{45}$$

Substituting (43), (44) and (45) in the equations (38), (39), (40), (41) and (42) and by equating the coefficient of λ^0, λ^1 and λ^2, and neglecting the terms of order greater than or equal to $O\left(\lambda^3\right)$, the following ordinary differential equations are obtained:

$$\psi_0{}^{iv}-M\psi_0''=G_r\theta_0'+G_m\phi_0' \tag{46}$$

$$\psi_1{}^{iv}-iu_0\psi_0''-M\psi_1''+iu_0''\psi_0=G_r\theta_1'+G_m\phi_1' \tag{47}$$

$$\psi_2{}^{iv}-M\psi_2''-iu_0\psi_1''+iu_0''\psi_1-2\psi_0''=G_r\theta_2'+G_m\phi_2' \tag{48}$$

$$\theta_0{}^{iv}-P_rN\theta_0+P_rD_u\phi_0''=0 \tag{49}$$

$$\theta_1{}^{iv}-P_riu_0\theta_0-P_rN\theta_1+P_rD_u\phi_1''=P_ri\psi_0T_0' \tag{50}$$

$$\theta_2{}^{iv}-\theta_0-P_rN\theta_2-P_ru_0i\theta_1-P_rD_u\phi_0+P_rD_u\phi_2''=P_ri\psi_1T_0' \tag{51}$$

$$\phi_0''+S_cS_r\theta_0''=0 \tag{52}$$

$$\phi_1''+S_cS_r\theta_1''=iS_cu_0\phi_0+iS_cC_0'\psi_0 \tag{53}$$

$$\phi_2''-\phi_0+S_cS_r\left(\theta_2''-\theta_0\right)=iS_cu_0\phi_1+iS_cC_0'\psi_1 \tag{54}$$

Subject to the following boundary conditions:

$$\left.\begin{array}{lllll}\psi_0'(y)=u_0'(0), & \psi_0(y)=0, & \theta_0(y)=-T_0'(0), & \phi_0(y)=-C_0'(0) & \text{at } y=0\\ \psi_0'(y)=0, & \psi_0(y)=0, & \theta_0(y)=0, & \phi_0(y)=0 & \text{at } y=1\end{array}\right\} \tag{55}$$

$$\left.\begin{array}{lllll}\psi_1'(y)=0, & \psi_1(y)=0, & \theta_1(y)=0, & \phi_1(y)=0 & \text{at } y=0\\ \psi_1'(y)=0, & \psi_1(y)=0, & \theta_1(y)=0, & \phi_1(y)=0 & \text{at } y=1\end{array}\right\} \tag{56}$$

$$\left.\begin{array}{lllll}\psi_2'(y)=0, & \psi_2(y)=0, & \theta_2(y)=0, & \phi_2(y)=0 & \text{at } y=0\\ \psi_2'(y)=0, & \psi_2(y)=0, & \theta_2(y)=0, & \phi_2(y)=0 & \text{at } y=1\end{array}\right\} \tag{57}$$

The solutions of the equations (46) to (54) subject to the boundary conditions (55), (56) and (57) are obtained but not presented here for the sake of brevity.

SKIN FRICTION:

The viscous drag per unit area at any point in the fluid in terms of skin friction $\bar{\tau}_{xy}$ is given by

$$\bar{\tau}_{xy}=\mu\left(\frac{\partial\bar{u}}{\partial\bar{y}}+\frac{\partial\bar{v}}{\partial\bar{x}}\right)$$

The non dimensional skin friction τ_{xy} at any point is specified by:

$$\tau_{xy}=\frac{d^2\bar{\tau}_{xy}}{\rho v^2}=u_0'(y)+\varepsilon e^{i\lambda x}u_1'(y)+i\varepsilon\lambda e^{i\lambda x}v_1'(y)$$

At the wavy wall $y=\varepsilon\cos\lambda x,$ the co-efficient of skin friction is given by:

$$\tau_w = \left[\tau_{xy}\right]_{y=\varepsilon\cos\lambda x} = \tau_0^0 + \varepsilon \operatorname{Re}\left[e^{i\lambda x}u_0''(0) + e^{i\lambda x}u_1'(0)\right]$$

where $\tau_0^0 = u_0'(0)$

At the flat wall $y = 1$, the co-efficient of skin friction is determined by:

$$\tau_1 = \left[\tau_{xy}\right]_{y=1} = \tau_1^0 + \varepsilon \operatorname{Re}\left[e^{i\lambda x}u_1'(1)\right],$$

where $\tau_1^0 = u_0'(1)$

HEAT TRANSFER COEFFICIENT:

The non dimensional heat transfer co-efficient in terms of Nusselt number Nu is given by:

$$Nu = \frac{\partial T}{\partial y} = T_0'(y) + \varepsilon\frac{\partial T_1}{\partial y} = T_0'(y) + \varepsilon e^{i\lambda x}\theta'(y)$$

At the wavy wall $y = \varepsilon\cos\lambda x$, it is as under:

$$\begin{aligned} Nu_w = \left[\frac{\partial\theta}{\partial y}\right]_{y=\varepsilon\cos\lambda x} &= T_0'(\varepsilon\cos\lambda x) + \varepsilon e^{i\lambda x}\theta'(\varepsilon\cos\lambda x) \\ &= T_0'(0) + \varepsilon\cos\lambda x T_0''(0) + \varepsilon e^{i\lambda x}\left[\theta'(0) + \varepsilon\cos\lambda x\theta''(0)\right] \\ &= T_0'(0) + \varepsilon\cos\lambda x T_0''(0) + \varepsilon e^{i\lambda x}\theta'(0) \qquad \text{(neglecting } \varepsilon^2\text{)} \\ &= Nu_0^0 + \varepsilon \operatorname{Re}\left[e^{i\lambda x}T_0''(0) + e^{i\lambda x}\theta'(0)\right], \quad \text{where } Nu_0^0 = T_0'(0) \end{aligned}$$

At the flat wall $y = 1$, the Nusselt number is represented by:

$$\begin{aligned} Nu_1 = \left[\frac{\partial\theta}{\partial y}\right]_{y=1} &= T_0'(1) + \varepsilon e^{i\lambda x}\theta'(1) \\ &= Nu_1^0 + \varepsilon \operatorname{Re}\left[e^{i\lambda x}\theta'(1)\right], \quad \text{where } Nu_1^0 = T_0'(1) \end{aligned}$$

MASS TRANSFER CO-EFFICIENT:

The non dimensional mass transfer co-efficient in terms of Sherwood number Sh is given by:

$$Sh = \frac{\partial C}{\partial y} = C_0'(y) + \varepsilon\frac{\partial C_1}{\partial y} = C_0'(y) + \varepsilon e^{i\lambda x}\phi(y)$$

At the wavy wall $y = \varepsilon\cos\lambda x$, the Sherwood number is as follows:

$$\begin{aligned} Sh_w = \left[\frac{\partial C}{\partial y}\right]_{y=\varepsilon\cos\lambda x} &= C_0'(\varepsilon\cos\lambda x) + \varepsilon e^{i\lambda x}\phi'(\varepsilon\cos\lambda x) \\ &= C_0'(0) + \varepsilon\cos\lambda x C_0''(0) + \varepsilon e^{i\lambda x}\phi'(0) \qquad \text{(neglecting } \varepsilon^2\text{)} \\ &= Sh_0^0 + \varepsilon \operatorname{Re}\left[e^{i\lambda x}C_0''(0) + e^{i\lambda x}\phi'(0)\right], \quad \text{where } Sh_0^0 = C_0'(0) \end{aligned}$$

At the flat wall $y = 1$, the Sherwood number is defined by:

$$Sh_1 = \left[\frac{\partial C}{\partial y}\right]_{y=1} = C_0'(1) + \varepsilon e^{i\lambda x}\phi'(1) = Sh_1^0 + \varepsilon \operatorname{Re}\left[e^{i\lambda x}\phi'(1)\right],$$

where $Sh_1^0 = C_0'(1)$

The figures 1 to 6 represent the variations of the velocity u of the fluid versus y under the effects of radiation parameter N, Velocity of the flat wall U, Hartmann number M, Thermal Grashof number Gr, Solutal Grashof number Gm, and Heat source parameter α. From these figures it is noticed that the velocity fluid increases as U, Gr, Gm and α increases and decreases as N and M increases. This indicates a growth in the fluid motion owing to a rise in the velocity of the flat wall and under the effect of buoyancy forces and heat generating source whereas the fluid motion is retarded due to the imposition of the thermal radiation and the transverse magnetic field.

The behaviours of the temperature field T against y under the influence of the parameters N, Sr, Du and α are demonstrated through figures 7 to 10. It is obvious from these figures that the fluid temperature T is augmented as α is increased and it falls with a rise in each of N, Sr and Du. Hence, it is inferred that the fluid temperature falls down under the influence of the thermal radiation, thermal-diffusion and diffusion-thermo, but it rises up due to an increase in the strength of the heat generating source.

The variations of species concentration C versus y under the influence of radiation parameter N, Schmidt number Sc, Soret number Sr, and Dufour number Du are presented in figures 11 to 14. It is observed from these figures that the concentration level of the fluid is boosted on account of the increasing values of N, Sc, Sr and Du. In other words, the thickness of the concentration boundary layer decreases under the effect of mass diffusion whereas the thickness of the concentration boundary layer increases under the effects of thermal radiation, thermal-diffusion and diffusion-thermo.

The nature of skin friction τ at both the wavy wall and uniformly moving parallel flat wall is demonstrated in figures 15, 16 and 17. It is inferred that the magnitude of the viscous drag at the wavy wall exhibits an increase with an increase in the plate velocity U whereas it decreases at the flat wall as the plate velocity is raised. However, the magnitudes of the viscous drag at both the walls fall down under the influence of thermal radiation and Dufour effects. From all the figures 15 to 17, it is clear that the imposition of the transverse magnetic field (M) leads to a decrease in the magnitude of viscous drag at the wavy wall.

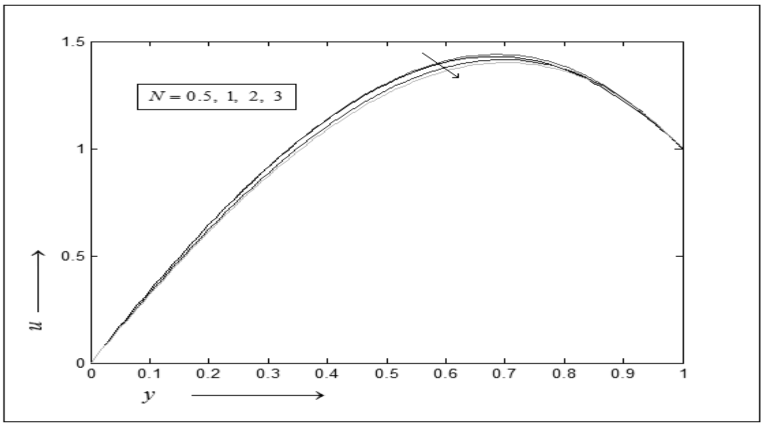


Figure 1: Velocity u versus y under N for $\Pr = .71,\ Du = .2,\ M = .5,\ Sc = .6,$

$Sr = 1,\ \alpha = 1,\ Gr = 2,\ Gm = 2,\ m = 5,\ n = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01,\ U = 1$

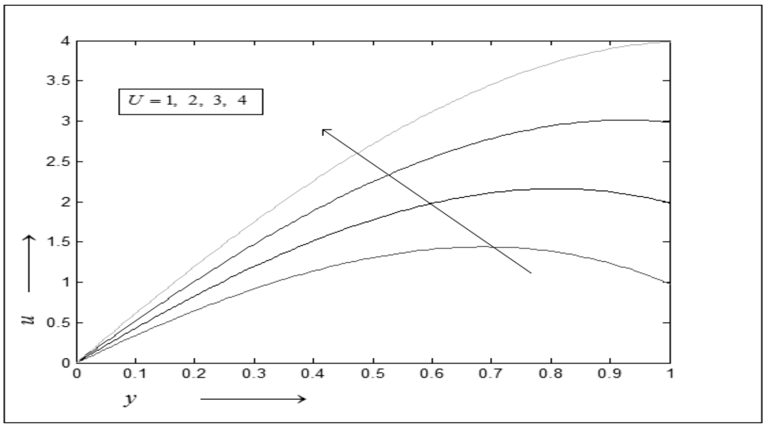


Figure 2: Velocity u versus y under U for $\Pr = .71,\ Du = .2,\ M = .5,\ Sc = .6,$

$Sr = 1,\ \alpha = 1,\ Gr = 2,\ Gm = 2,\ N = .5,\ m = 5,\ n = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01,$

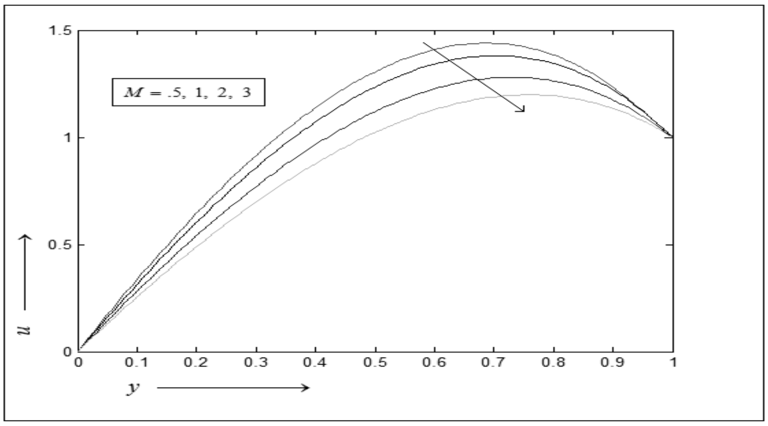


Figure 3: Velocity u versus y under M for $\Pr = .71,\ Du = .2,\ Sc = .6,\ Sr = 1,$

$U = 1,\ \alpha = 1,\ Gr = 2,\ Gm = 2,\ N = .5,\ m = 5,\ n = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$

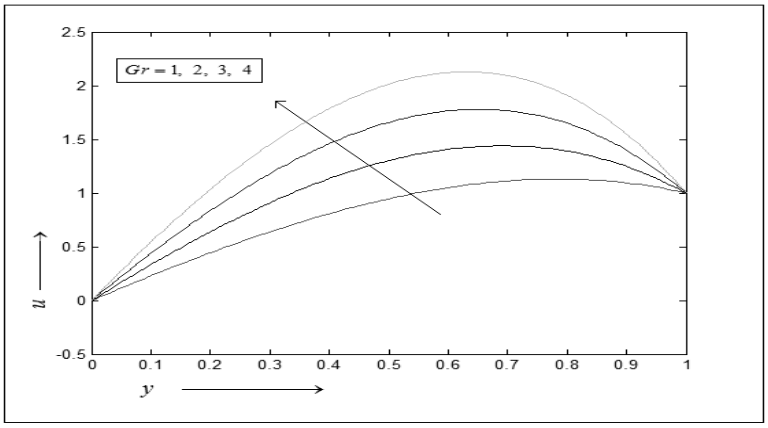


Figure 4: Velocity u versus y under Gr for $\Pr = .71,\ Du = .2,\ Sc = .6,\ Sr = 1,$

$$U = 1,\ \alpha = 1,\ M = .5,\ Gm = 2,\ N = .5,\ m = 5,\ n = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$$

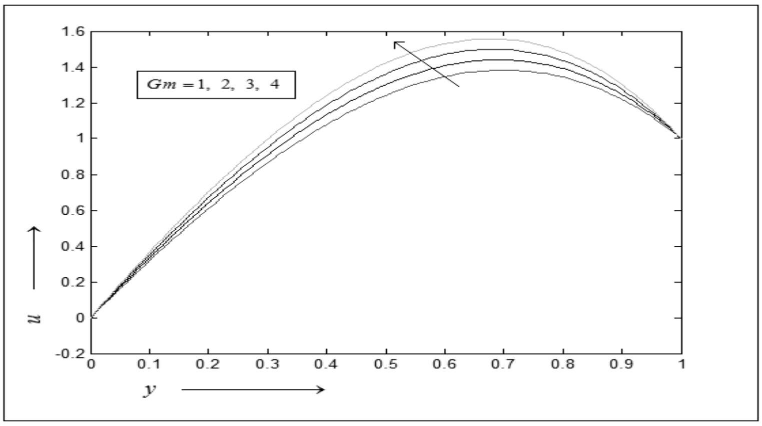


Figure 5: Velocity u versus y under Gm for $\Pr = .71,\ Du = .2,\ Sc = .6,\ Sr = 1,$

$$U = 1,\ \alpha = 1,\ M = .5,\ Gr = 2,\ N = .5,\ m = 5,\ n = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$$

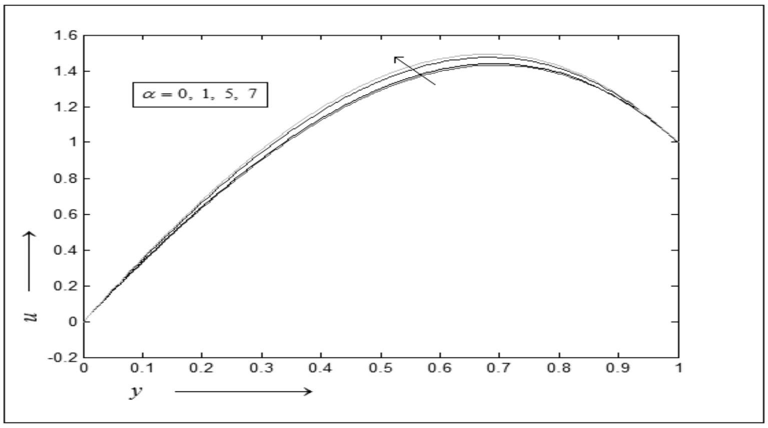


Figure 6: Velocity u versus y under α for $\Pr = .71,\ Du = .2,\ Sc = .6,\ Sr = 1,$

$$U = 1,\ M = .5,\ Gr = 2,\ Gm = 2,\ N = .5,\ m = 5,\ n = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$$

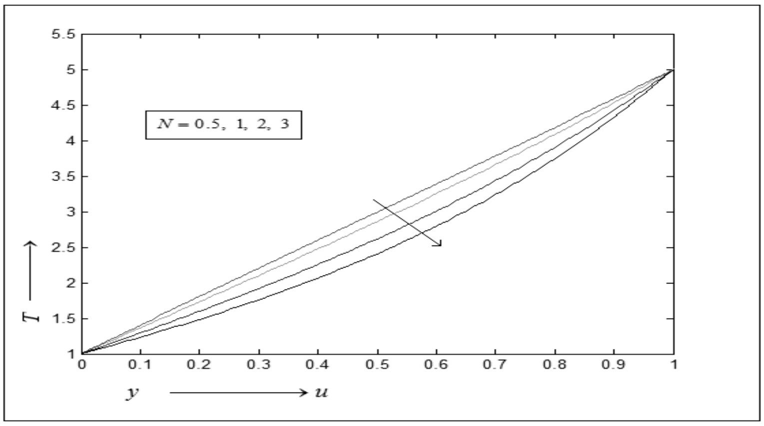


Figure 7: Temperature T versus y under N for $\Pr=.71,\ Du=.2,\ Sc=.6,\ Sr=1,$ $\alpha=1,\ U=1,\ M=.5,\ Gr=2,\ Gm=2,\ m=5,\ n=1,\ \lambda x=\dfrac{\pi}{2},\ \lambda=.001,\ \varepsilon=.01$

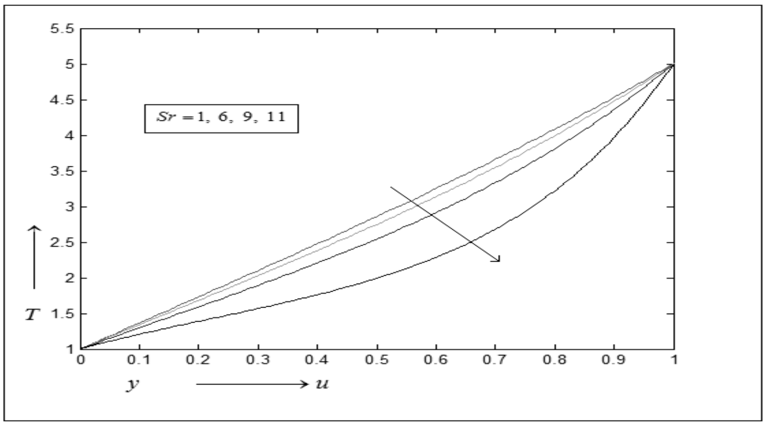


Figure 8: Temperature T versus y under Sr for $\Pr=.71,\ Du=.2,\ Sc=.6,\ \alpha=1,$ $U=1,\ M=.5,\ Gr=2,\ Gm=2,\ m=5,\ n=1,\ N=1,\ \lambda x=\dfrac{\pi}{2},\ \lambda=.001,\ \varepsilon=.01$

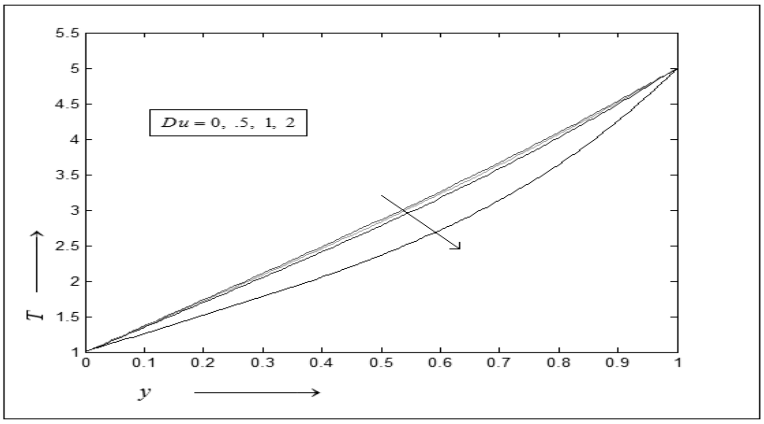


Figure 9: Temperature T versus y under Du for $\Pr=.71,\ Sc=.6,\ Sr=1,\ \alpha=1,$ $U=1,\ M=.5,\ Gr=2,\ Gm=2,\ m=5,\ n=1,\ N=1,\ \lambda x=\dfrac{\pi}{2},\ \lambda=.001,\ \varepsilon=.01$

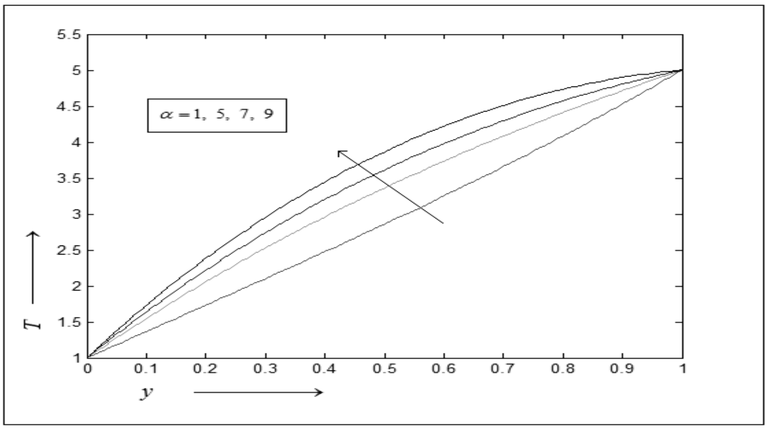


Figure 10: Temperature T versus y under α for $\Pr = .71,\ Du = .2,\ Sc = .6,\ Sr = 1,$ $U = 1,\ M = .5,\ Gr = 2,\ Gm = 2,\ m = 5,\ n = 1,\ N = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$

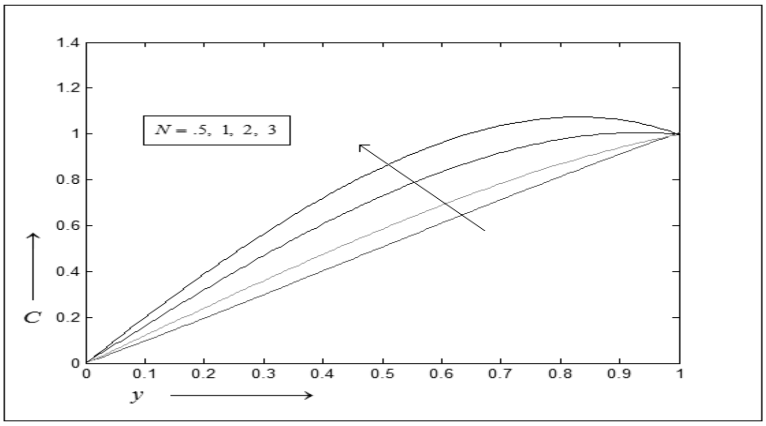


Figure 11: Concentration C versus y under N for $\Pr = .71,\ Du = .2,\ Sc = .6,\ Sr = 1,$ $\alpha = 1,\ U = 1,\ M = .5,\ Gr = 2,\ Gm = 2,\ m = 5,\ n = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$

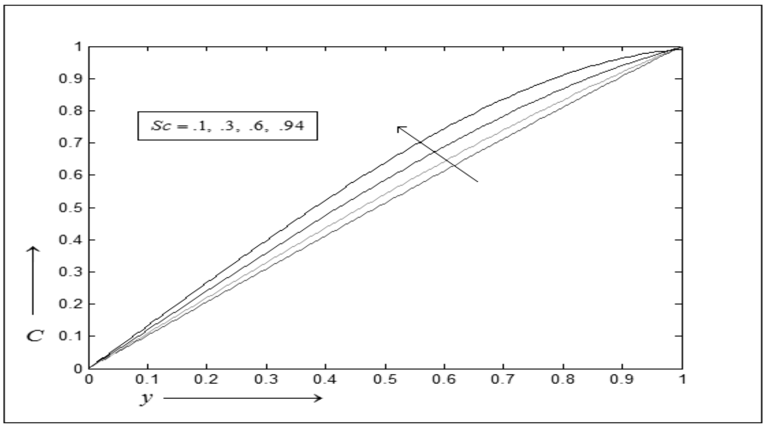


Figure 12: Concentration C versus y under Sc for $\Pr = .71,\ Du = .2,\ Sr = 1,\ \alpha = 1,$ $U = 1,\ M = .5,\ Gr = 2,\ Gm = 2,\ m = 5,\ n = 1,\ N = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$

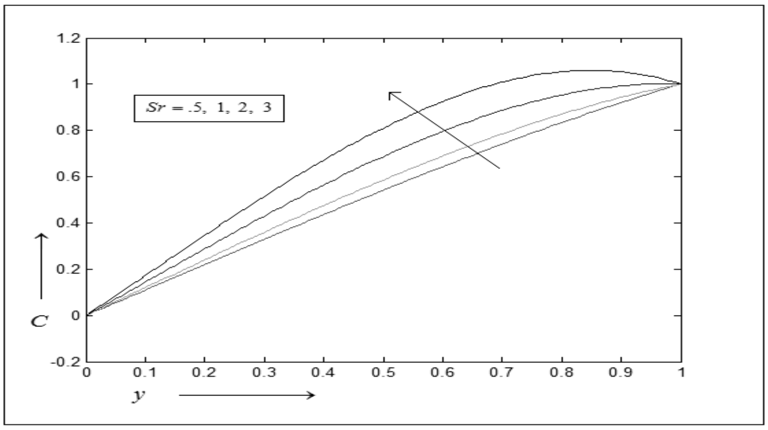


Figure 13: Concentration C versus y under Sr for $\Pr = .71,\ Du = .2,\ Sc = .6,\ \alpha = 1,$ $U = 1,\ M = .5,\ Gr = 2,\ Gm = 2,\ m = 5,\ n = 1,\ N = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$

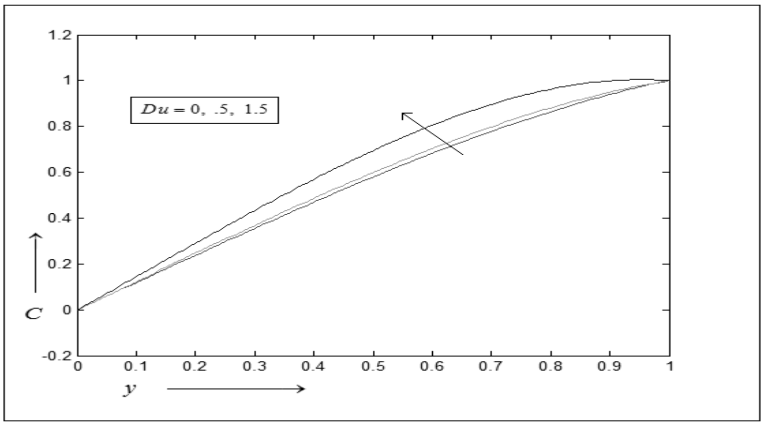


Figure 14: Concentration C versus y under Du for $\Pr = .71,\ Sc = .6,\ Sr = 1,\ \alpha = 1,$ $U = 1,\ M = .5,\ Gr = 2,\ Gm = 2,\ m = 5,\ n = 1,\ N = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$

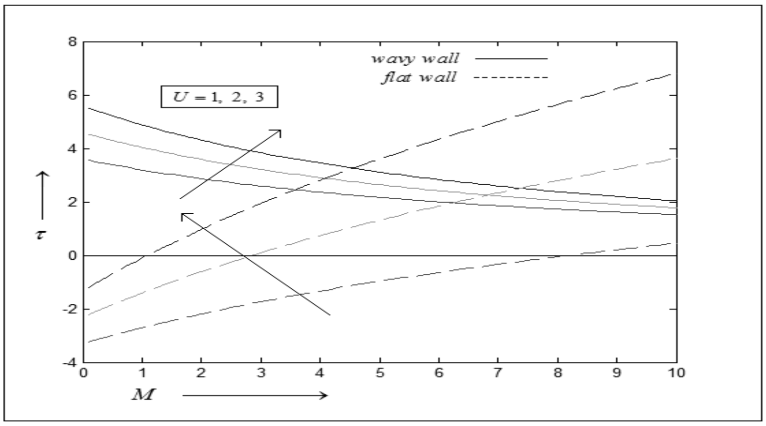


Figure 15: Skin friction τ versus M under U for $\Pr = .71,\ Du = .2,\ Sc = .6,\ Sr = 1,$ $\alpha = 1,\ Gr = 2,\ Gm = 2,\ m = 5,\ n = 1,\ N = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$

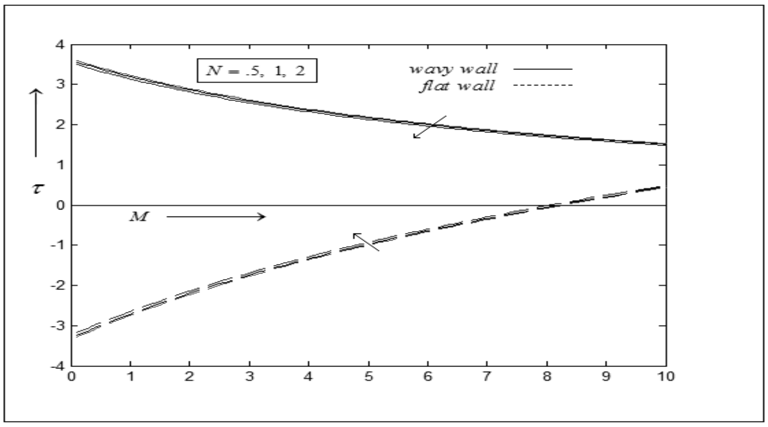


Figure 16: Skin friction τ versus M under N for $\Pr = .71,\ Du = .2,\ Sc = .6,\ Sr = 1,$

$\alpha = 1,\ Gr = 2,\ Gm = 2,\ m = 5,\ n = 1,\ U = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$

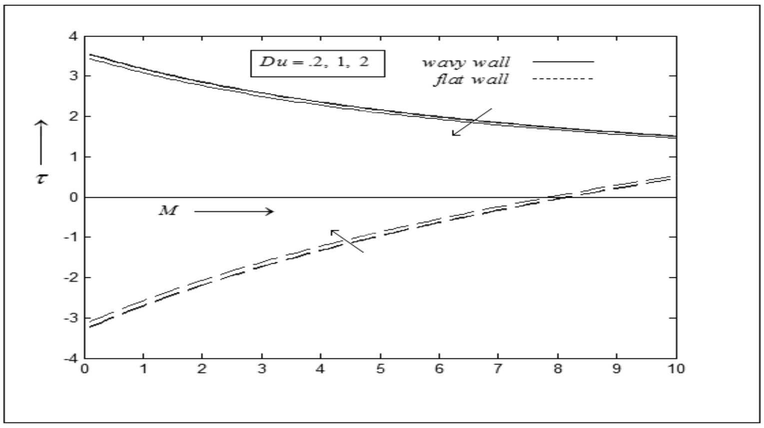


Figure 17: Skin friction τ versus M under Du for $\Pr = .71,\ Sc = .6,\ Sr = 1,\ \alpha = 1,$

$Gr = 2,\ Gm = 2,\ m = 5,\ n = 1,\ N = 1,\ U = 1,\ \lambda x = \frac{\pi}{2},\ \lambda = .001,\ \varepsilon = .01$

CONCLUSIONS:

- The fluid motion is retarded under the application of thermal radiation and the transverse magnetic field and accelerated under the effect of plate velocity.
- An increase in each of radiation parameter, Soret number and Dufour number leads the fluid temperature to fall.
- The thickness of the concentration boundary layer increases under thermal radiation, Soret and Dufour effects.
- The thermal radiation or diffusion-thermo effect leads to a fall in the magnitude of the viscous drag at the wavy wall as well as at the flat wall.
- Finally, it may be concluded that the radiation effect, Soret effect and Dufour effect play a significant role in controlling the flow and transport characteristics, under the present model for flow, heat and mass transfer.

NOMENCLATURE:

K_λ	Absorption co-efficient
g	Acceleration due to gravity
ε	Amplitude parameter
$\bar{x}$, $\bar{y}$	Cartesian co-ordinates
μ	Coefficient of viscosity
β	Coefficient of volume expansion for heat transfer
C_s	Concentration susceptibility
D_M	Coefficient of mass diffusion
Q	Constant heat addition / absorption
d	Distance between two walls
ρ_s	Density of the fluid in static condition
D_u	Dufour number
σ	Electrical conductivity
$\bar{p}$	Fluid pressure
ρ	Fluid density
λ	Frequency parameter
$\bar{T}$	Fluid temperature
$\bar{T}_s$	Fluid temperature in static condition
G_r	Grashof number for heat transfer
G_m	Grashof number for mass transfer
α	Heat source parameter
υ	Kinematic viscosity
T_m	Mean fluid temperature
M	Magnetic parameter
$e_{b\lambda}$	Plank function
$\bar{p}_s$	Pressure of the fluid in static condition
P_r	Prandtl number
N	Radiation parameter
$\bar{B}$	Strength of the applied magnetic field
$\bar{C}$	Species concentration
C_p	Specific heat at constant pressure
$\bar{C}_w$	Species concentration at the wavy wall
$\bar{C}_1$	Species concentration at the flat wall
S_r	Soret number
S_c	Schmidt number
k	Thermal conductivity

$\bar{T}_w$ Temperature of the wavy wall
$\bar{T}_1$ Temperature of the flat wall
K_T Thermal diffusion ratio
$\bar{u}$, $\bar{v}$ Velocity components
$\bar{\beta}$ Volumetric coefficient of the expansion with species concentration
U Velocity of the flat wall
m Wall temperature ratio
n Wall concentration ratio

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