

## **Variable Viscosity on Convective Heat Transfer in a Dusty Fluid over a Vertical Permeable Surface with Thermal Radiation**

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### **Abstract**

The effects variable viscosity on a steady free convective boundary layer flow of a dusty fluid past a vertical permeable stretching surface is studied. The system of non linear partial differential equations governing the equations motion are reduced to a system of ordinary differential equations using similarity transformations. The resultant boundary value problems are then solved numerically using shooting technique based on fourth order Runge-Kutta method. The effects of physical parameters viz., viscosity parameter, fluid-particle interaction parameter, local Grashof number, suction parameter, Prandtl number, radiation parameter and Eckert number on the flow and heat transfer are computed and presented graphically. The skin friction coefficient and the temperature gradient which gives the rate of heat transfer at the surface are also obtained and present in Tables. The effects of all the parameters are quite significant.

**Keywords:** Variable viscosity, Dusty fluid; free convection; fluid-particle interaction parameter; radiation parameter; suction parameter.

### **1. Introduction**

The study of laminar flow and heat transfer over a stretching sheet has considerable interest because of its ever increasing industrial applications and important bearings on several technological processes. It plays an important role in many industries such as chemical industry, power and cooling industry for drying, chemical vapour deposition on surfaces and cooling of nuclear reactors etc. Such processes occur when the effect

of buoyancy forces in free convection becomes significant. The problem of free convection under the influence of magnetic field has attracted numerous researchers in view of its applications in geophysics and astrophysics. A wide variety of problems dealing with heat and fluid flow over a stretching sheet have been studied for both Newtonian and non-Newtonian fluids under the influence of imposed electric and magnetic fields, under different thermal boundary conditions have been studied by many authors as reported in the literature. Crane [1] was the first to examine the problem of steady two-dimensional boundary layer flow of an incompressible and viscous fluid caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point. Grubka and Bobba [2] analyzed heat transfer effects by considering the power-law variation of surface temperature.

The flow and heat transfer over a vertical stretching sheet under various physical situations have been reported by many researchers like Daskalakis [3], Chen [4], Chamkha [5], Ishak et al. [6], Saleh et al. [7], Gireesha et al. [8], Kumari et al. [9] have studied the unsteady free convection flow over a continuous moving vertical surface in an ambient fluid by taking two different heating processes like constant surface temperature (CST) and constant surface heat flux (CHF). Chakrabarti [10] studied the boundary layer flow of a dusty gas. Datta and Mishra [11] have investigated boundary layer flow of a dusty fluid over a semi-infinite flat plate. Several investigators such as Agranat [12], Vajravelu et al. [13], Evgeny et al. [14], XIE Ming-liang et al. [15] and Palani et al. [16] studied flow problems of dusty fluids. Gireesha et al. [17] obtained the numerical solution for boundary layer flow and heat transfer of a dusty fluid over a stretching sheet with non-uniform heat source/sink. The above investigations deal with the flow and heat transfer induced by a horizontal stretching sheet, but there arise some situations where the stretching sheet moves vertically in a cooling liquid. Under such circumstances the fluid flow and the heat transfer characteristics are determined by two mechanisms, namely motion of the stretching sheet and the buoyant force. Also, in the above studies, the radiation effect was ignored. But, in the systems operating at high temperatures, the thermal radiation heat transfer becomes very important and its effects cannot be neglected. Thermal radiation plays a significant role in controlling heat transfer processes in polymer industry.

Chen [18] analyzed MHD mixed convection of a power law fluid past a stretching surface in the presence of thermal radiation and internal heat generation/absorption. Mukhopadhyay and Layek [19] studied free convective flow and radiative heat transfer of a viscous incompressible fluid with variable viscosity over a stretching porous vertical plate. Ramesh et al. [20] studied the effects of thermal radiation on a steady boundary layer flow and heat transfer of an incompressible viscous dusty fluid over a permeable vertical stretching surface.

In all the earlier investigations viscosity is assumed as constant. But viscosity varies with temperature. In this paper an attempt has been made to study numerically the effects of variable viscosity on flow and heat transfer of an incompressible viscous dusty fluid over a permeable vertical stretching surface with thermal radiation. The resulting non-linear boundary value problem in ordinary differential equations is solved by shooting method based on fourth order Runge-Kutta method. Velocity and temperature distributions are presented in graphs for various flow characteristic

parameters. The skin friction coefficient which gives the wall shear stress and temperature gradient at the surface which is the rate of heat transfer are presented in tables. The effects of all the flow parameters are significant.

## 2. Flow analysis of the problem

Consider a steady two dimensional laminar boundary layer flow of an incompressible viscous dusty fluid over a vertical stretching sheet. The flow is generated by the action of two equal and opposite forces along the  $x$ -axis and  $y$ -axis being normal to the flow. The sheet being stretched with a velocity  $U_w(x)$  along the  $x$ -axis, keeping the origin fixed in the fluid of ambient temperature  $T$ .

The dust particles are assumed to be spherical in shape and uniform in size and number density of the dust particle is taken as a constant throughout the flow. With these assumptions and usual boundary layer approximations, and following Vajravelu and Nayfeh [6] we get the equations motions as :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{KN}{\rho} (u_p - u) + \square \beta^* (T - T_\infty) \quad (2)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{K}{m} (u - u_p) \quad (3)$$

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{K}{m} (v - v_p) \quad (4)$$

$$\frac{\partial}{\partial x} (\rho_p u_p) + \frac{\partial}{\partial y} (\rho_p v_p) = 0 \quad (5)$$

where  $(u, v)$  and  $(u_p, v_p)$  are the velocity components of the fluid and dust particle phases along  $x$  and  $y$  directions respectively.  $\mu, \rho, \rho_p$  and  $N$  are the co-efficient of viscosity of the fluid, density of the fluid, density of the particle phase, number density of the particle phase respectively.  $K$  is the stokes resistance (drag co-efficient).  $T$  and  $T_\infty$  are the fluid temperature with in the boundary layer and in the free stream respectively.  $g$  is the acceleration due to gravity,  $\beta^*$  is the volumetric coefficient of thermal expansion,  $m$  is the mass of the dust particle respectively. In deriving these equations, the drag force is considered for the interaction between the fluid and particle phases.

The dusty boundary layer heat transport equations with radiation are:

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \frac{N c_p}{\tau_T} (T_p - T) + \frac{N}{\tau_v} (u_p - u)^2 - \frac{\partial q_r}{\partial y} \quad (6)$$

$$u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = - \frac{c_p}{c_m \tau_T} (T_p - T) \quad (7)$$

where  $T$  and  $T_p$  is the temperature of the fluid and temperature of the dust particle,  $c_p$  and  $c_m$  are the specific heat of fluid and dust particles,  $\tau_r$  is the thermal equilibrium time and is the time required by the dust cloud to adjust its temperature to the fluid,  $\tau_v$  is the relaxation time of the dust particle i.e., the time required by a dust particle to adjust its velocity relative to the fluid,  $k$  is the thermal conductivity.

Using the Rosseland approximation for radiation [4], radiation heat flux is simplified as

$$q_r = -\frac{4\sigma^*}{3k^*} \partial T^4 / \partial y \quad (8)$$

where  $\sigma^*$  and  $k^*$  are the Stefan-Boltzman constant and the mean absorption co-efficient respectively. The temperature differences within the flow is such that the term  $T^4$  may be expressed as a linear function of the temperature. Expanding  $T^4$  in a Taylor series about  $T_\infty$  and neglecting the higher order terms beyond the first degree in  $(T - T_\infty)$  we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (9)$$

Substituting equations (8) and (9) in (6) we get,

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \left( k + \frac{16\sigma^* T_\infty^3}{3k^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{N c_p}{\tau_r} (T_p - T) + \frac{N}{\tau_v} (u_p - u)^2 \quad (10)$$

The boundary conditions for the flow problem are given by

$$u = U_W(x), v = -V_W(x), T = T_w = T_\infty + A \left( \frac{x}{l} \right)^2 \text{ at } y=0 \quad (11)$$

$$u \rightarrow 0, u_p \rightarrow 0, v_p \rightarrow v, \rho_p \rightarrow \omega \rho, T \rightarrow T_\infty, T_p \rightarrow T_\infty \text{ as } y \rightarrow \infty \quad (18)$$

where  $U_W(x) = cx$  is the stretching sheet velocity,  $c > 0$  is the stretching rate,  $\omega$  is the density ratio,  $T_w$  and  $T_\infty$  denote the temperature at the wall and at large distance from the wall respectively,  $A$  is a positive constant and  $l = \sqrt{\frac{\nu}{c}}$  is a characteristic length.

Following Lai and Kulacki [21] we assume that the viscosity and thermal conductivity are inverse linear functions of temperature, i.e.

$\frac{1}{\mu} = \frac{1}{\mu_\infty} [1 + \delta(T - T_\infty)]$ , or  $\frac{1}{\mu} = a(T - T_r)$  where  $a = \frac{\delta}{\mu_\infty}$  and  $T_r = T_\infty - \frac{1}{\delta}$  where  $a$  and  $T_r$  are constants and their values depend on the reference state i.e.  $\nu$ . In general  $a > 0$  for liquids and  $a < 0$  for gases.

Let us introduce the following similarity transformations,

$$u = cx f'(\eta), v = -\sqrt{\nu c} f(\eta), \eta = \sqrt{\frac{c}{\nu}} y$$

$$u_p = cx F(\eta), v_p = \sqrt{vc} G(\eta), \rho_r = H(\eta) \quad (13)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty} \quad (14)$$

$$\text{Where } T - T_\infty = A \left(\frac{x}{l}\right)^2 \theta(\eta)$$

Using (13) and (14) in (2) to (12), we obtain the following non-linear ordinary differential equations

$$\frac{\theta_r}{\theta - \theta_r} f''' + f f'' - [f'^2 - Gr \theta + l^* \beta H [F - f']] = 0 \quad (15)$$

$$G F' + F^2 + \beta [F - f'] = 0 \quad (16)$$

$$G G' + \beta [f + G] = 0 \quad (17)$$

$$H F + H G' + G H' = 0 \quad (18)$$

$$(1 + N_r) \theta'' + P_r [f \theta' - 2f' \theta] + \frac{N_r P_r}{\rho_r c \tau} [\theta_p - \theta] + N_r Pr Ec [F - f']^2 = 0, \dots (19)$$

$$2F \theta_p + G \theta_p' + \frac{c_p}{cc_m \tau T} [\theta_p - \theta] = 0, \quad (20)$$

where a prime denotes differentiation with respect to  $\eta$  and  $l^* = \frac{mN}{p}$ ,  $\tau = \frac{m}{K}$  is the relaxation time of the particle phase,  $\beta = \frac{1}{c\tau}$  is the fluid particle interaction parameter,  $Gr = \frac{g\beta^*(T_w - T_\infty)}{c^2 x}$  is the local Grashof number,  $\rho_r = \frac{p_p}{p}$  is the relative density,  $Pr = \frac{\mu c_p}{k^*}$  is the Prandtl number,  $Ec = \frac{cl^2}{Ac_p}$  is the Eckert number,  $N_r = \frac{16\sigma^* T_\infty^3}{3kk^*}$  is the Radiation parameter and  $\theta_r = \left(\frac{T_r - T_\infty}{T_w - T_\infty}\right)$  is the viscosity parameter.

The boundary conditions (11) and (12) become

$$f = f_0, f' = 1, \theta = 1 \text{ at } \eta = 0, \quad (21)$$

$$f' = 0, F = 0, G = -f, H = \omega, \theta \rightarrow 0, \theta_p \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (22)$$

where  $f_0 = \frac{v_w}{(vc)^{1/2}}$  is the suction parameter.

The physical quantities of interest are the skin friction coefficient  $c_f$ , and the Nusselt number  $Nu$  which indicate physically wall shear stress and rate of heat transfer respectively. The wall shear stress  $\tau_w$  is given by

$$\tau_w = \left[ (\mu) \frac{\partial u}{\partial y} \right]_{y=0} = \rho v_\infty \left( \frac{\theta_k}{\theta_c - 1} \right) \sqrt{\frac{U_0^3 \alpha^2}{v_\infty c_0}} f''(0) \quad (23)$$

The skin –friction coefficient  $c_f$  can be defined as

$$c_f = \frac{2 \tau_w}{\rho U_0^2} = 2 \left( \frac{\theta_c}{\theta_c - 1} \right) f''(0) \quad (24)$$

The heat transfer from the plate is given by

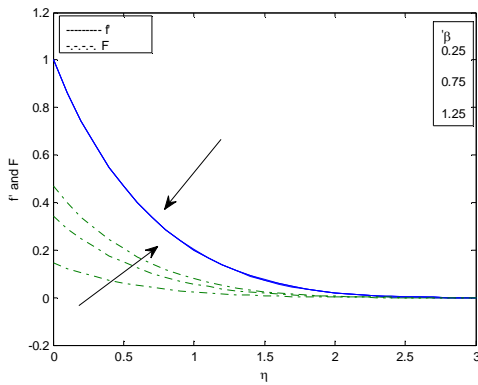
$$q_w = -k \left[ \frac{\partial T}{\partial y} \right]_{y=0} = -k \sqrt{\frac{U_0 c_0}{v_\infty \alpha^2}} \frac{(T_w - T_\infty) \alpha}{U_0} \theta'(0)$$

The Nusselt number is given by

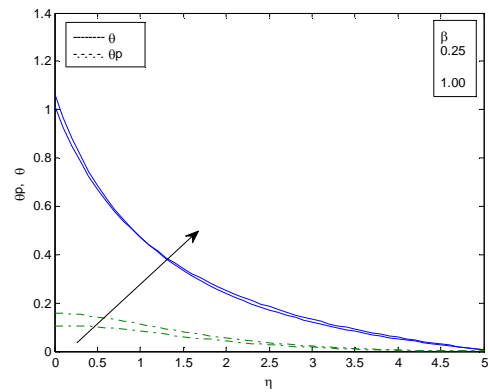
$$Nu = \frac{q_w U_0}{k(T_w - T_\infty) \alpha} = -\theta'(0) \quad (25)$$

### 3. Results and discussion

The equations (15) to (20) together with the boundary conditions (21) and (22) are solved numerically using fourth order Runge-Kutta shooting method. Numerical values are obtained for different values of parameters as shown in graphs and tables for  $l=1$ ,  $\beta=0.51$ ,  $\omega=0.1$ ,  $Pr=1.0$ ,  $Nr=3$ ,  $Ec=1$ ,  $fo=0.30$ ,  $Gr=5$ ,  $K1=1$  and  $\theta_r=-10$ .



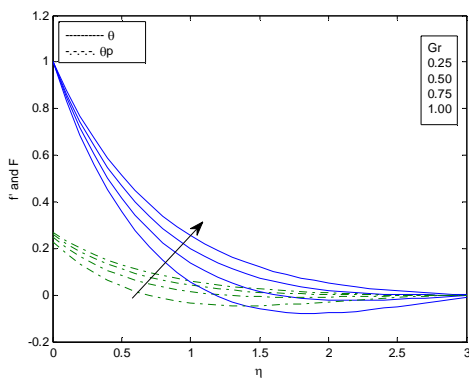
**Fig. 1:** Velocity profiles for different values of  $\beta$ .



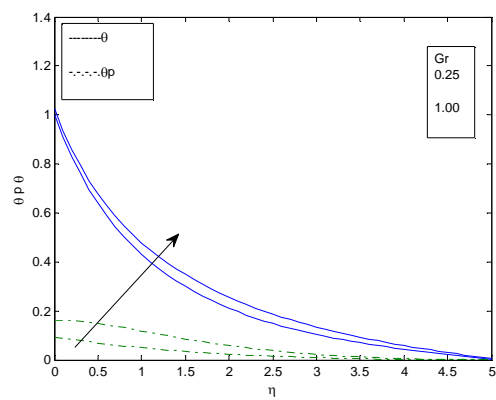
**Fig. 2:** Temperature profiles for different values of  $\beta$ .

Fig. 1 and Fig. 2, represent the graphs of velocity and temperature distribution respectively for various values of fluid particle interaction parameter ( $\beta$ ). It is observed from figures that if  $\beta$  increases there is slight decrease in the fluid phase velocity but significant increase in the dust phase velocity. Temperature increase in both fluid and dust phase for increasing value of fluid interaction parameter  $\beta$ .

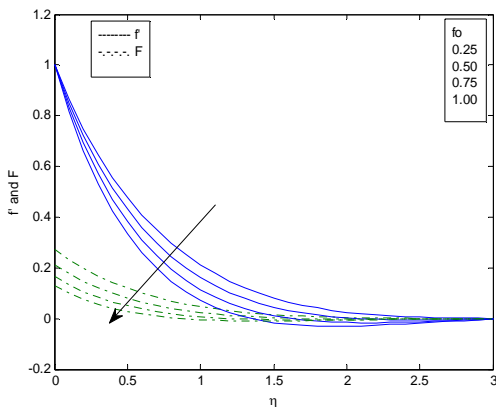
Fig. 3, gives the of velocity profiles for various values of local Grashof number ( $Gr$ ). It is observed that for increasing values of local Grashof number enhance the velocity of both the fluid and dust phases. Physically  $Gr > 0$  means heating of the fluid or cooling of the boundary surface,  $Gr < 0$  means cooling of the fluid or heating of the boundary surface and  $Gr=0$  corresponds to the absence of free convection current. Fig. 4, depicts the effect of local Grashof number ( $Gr$ ) on temperature profiles. It is seen that increasing value of  $Gr$  results in thinning of the thermal boundary layer associated with an increase in the wall temperature gradient and hence produces an increase in the heat transfer rate.



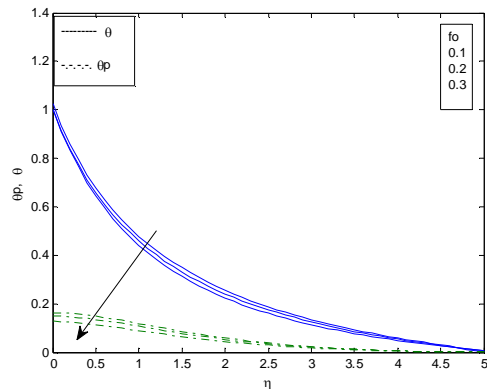
**Fig. 3:** Velocity profiles for different values of  $Gr$



**Fig. 4:** Temperature profiles for different values of  $Gr$ .



**Fig. 5:** Velocity profiles for different values of  $f_0$ .

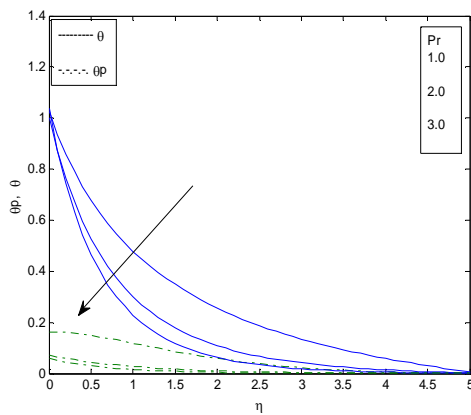


**Fig. 6:** Temperature profiles for different values of  $f_0$ .

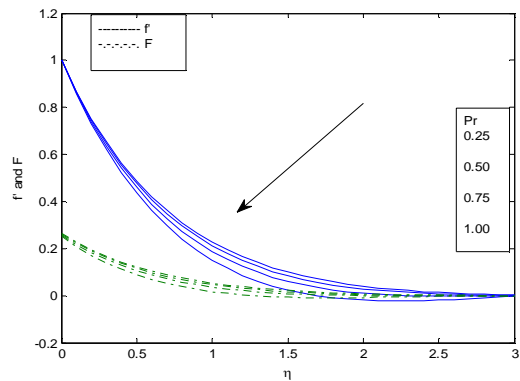
Fig. 5, shows the effect of suction parameter  $f_0$  on the velocity profiles. It is observed that both fluid and dust phase velocity profiles tends asymptotically to zero. The velocity profiles decreases as suction parameter  $f_0$  increases. It is seen from the Fig. 6 that the

fluid and dust phase temperature profiles decrease monotonically for increase of suction.

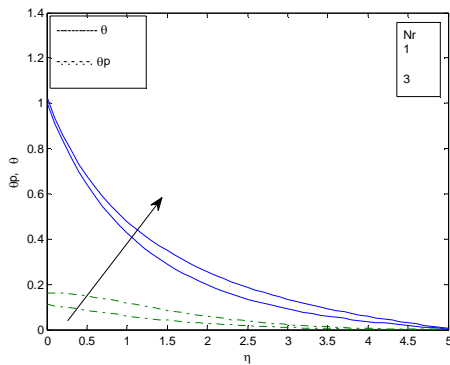
Fig. 7, gives the graph of temperature profile  $\theta(\eta)$  for various values of Prandtl number  $Pr$ . It is seen that the fluid phase temperature and dust phase temperature decrease with increase of Prandtl number. Velocity distribution for various values of  $Pr$  is plotted in Fig. 8. It is observed that for increasing values of Prandtl number  $Pr$  retarded the velocity for both the fluid phase as well as dust phase. It is seen from all the figures that the fluid phase is almost parallel to the dust phase and velocity and temperature in the fluid phase is higher than that of the dust phase.



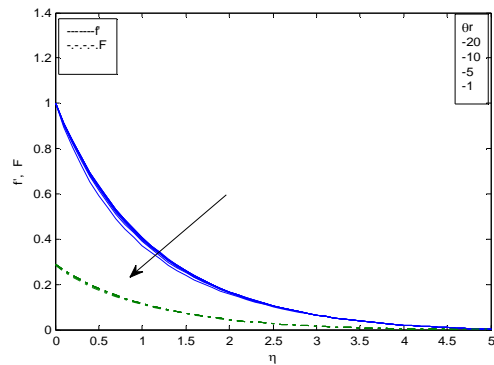
**Fig. 7:** Temperature profiles for different values of  $Pr$ .



**Fig. 8:** Velocity profiles for different values of  $Pr$ .



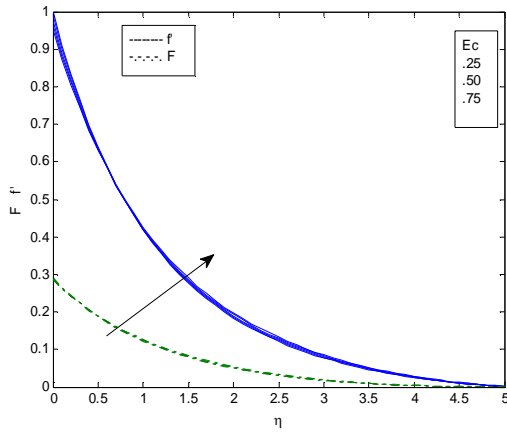
**Fig. 9:** Temperature profiles for different values of  $Nr$ .



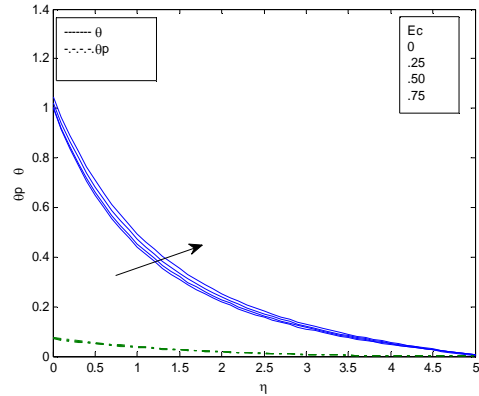
**Fig. 10:** Velocity profiles for different values of  $\theta_r$ .

The temperature profiles  $\theta(\eta)$  and  $\theta_p(\eta)$  are plotted for different values of radiation parameter  $Nr$  in Fig. 9. It is observed that the increase in the thermal radiation parameter  $Nr$  enhance temperature significantly.





**Fig. 11:** Velocity profiles for different values of  $Ec$



**Fig. 12:** Temperature profiles for different values of  $Ec$ .

The temperature and velocity profiles for different values of Eckert number  $Ec$  is presented in figures 11 and 12. The Eckert number show no significant effects on the velocity profiles where as it enhances the temperature.

Finally the skin friction coefficient  $C_f$  and rate of heat transfer given by Nusselt number  $Nu$  are shown in the tables (1)—(4) for different values of  $Ec$ ,  $Nr$ ,  $Gr$  and  $Pr$  against  $\theta_r$  and for  $l=1$ ,  $\beta=.51$ ,  $\omega=0.1$ ,  $Pr=1.$ ,  $Nr=3$ ,  $Ec=.1$ ,  $fo=.30$ ,  $Gr=.5$  and  $K1=1$ . The behaviour of these parameters is self evident from the tables and hence any further discussions about them seem to be redundant.

Values of skin friction coefficient  $C_f$  and Nusselt number  $Nu$

**Table 1**

$\theta_r$	$Ec \rightarrow 0.2$		$0.4$		$0.6$		$0.8$		$1.0$	
	$C_f$	$Nu$	$C_f$	$Nu$	$C_f$	$Nu$	$C_f$	$Nu$	$C_f$	$Nu$
-6.00	0.0104	0.0413	0.0107	0.0431	0.0110	0.0452	0.0114	0.0475	0.0117	0.0499
-4.00	0.0098	0.0411	0.0101	0.0429	0.0103	0.0448	0.0106	0.0470	0.0109	0.0494
-2.00	0.0082	0.0406	0.0084	0.0422	0.0086	0.0441	0.0089	0.0459	0.0090	0.0483

**Table 2**

$\theta_r$	$Nr \rightarrow 2$		$3$		$4$		$5$	
	$C_f$	$Nu$	$C_f$	$Nu$	$C_f$	$Nu$	$C_f$	$Nu$
-6.00	0.0131	0.0637	0.0103	0.0406	0.0091	0.0320	0.0085	0.0275
-4.00	0.0123	0.0630	0.0097	0.0404	0.0086	0.0319	0.0079	0.0274
-2.00	0.0102	0.0612	0.0081	0.0399	0.0072	0.0316	0.0067	0.0273

**Table 3**

Gr →	.2		0.4		0.6		0.8		1.0	
$\theta_r$										
↓	$C_f$	Nu	$C_f$	Nu	$C_f$	Nu	$C_f$	Nu	$C_f$	Nu
-6.00	0.0065	0.0397	0.0092	0.0403	0.0113	0.0409	0.0130	0.0414	0.0128	0.0421
-4.00	0.0061	0.0395	0.0086	0.0401	0.0106	0.0407	0.0125	0.0408	0.0139	0.0434
-2.00	0.0051	0.0391	0.0073	0.0397	0.0090	0.0401	0.0117	0.0396	0.0191	0.0381

**Table 4**

Pr →	0.2		0.4		0.6		0.8		1.0	
$\theta_r$										
↓	$C_f$	Nu	$C_f$	Nu	$C_f$	Nu	$C_f$	Nu	$C_f$	Nu
-6.00	0.0217	0.1333	0.0172	0.0957	0.0141	0.0706	0.0120	0.0531	0.0103	0.0406
-4.00	0.0206	0.1327	0.0162	0.0952	0.0133	0.0703	0.0112	0.0529	0.0097	0.0404
-2.00	0.0181	0.1312	0.0138	0.0940	0.0112	0.0694	0.0094	0.0522	0.0081	0.0399

#### 4. Conclusions

In this paper effects of viscosity and thermal radiation on flow and the free convective heat transfer of a dusty fluid over a vertical permeable stretching sheet is studied numerically. Velocity and temperature profiles are presented graphically. Influence of all the physical parameters found to effect the problem under consideration are the fluid particle interaction parameters viz., local Grashof number, suction parameter, radiation parameter, Prandtl number, Eckert number and viscosity parameter  $\theta_r$ . We may conclude as follows:

1. Velocity of fluid phase decreases and dust phase increases as  $\beta$  increases.
2. Velocity of fluid, dust phase increases and temperature of fluid and dust phase decreases as  $Gr$  increases.
3. Velocity and temperature of both fluid and dust phase decreases as  $Pr$  and  $f_0$  increases.
4. Temperature increases for increasing value of the radiation parameter  $Nr$  and Eckert number  $Ec$ .
5. Viscosity parameter  $\theta_r$  decreases the fluid and dust phase velocity.
6. Wall shear stress as well as the heat transfer rate increase with increase of viscosity, Prandtl number and thermal radiation but decrease with increase of Eckert number.

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