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Effects Of Viscous Dissipation, Radiation And Heat Source/Sink On Mhd Flow And Heat Transfer Over An Exponentially Stretching Sheet

Dr. V. Dhanalaxmi

Department of Mathematics, University College of Technology,
Osmania University,
Hyderabad, 500007, India.

ABSTRACT:

This paper presents a numerical solution for flow and heat transfer in the steady two dimensional laminar flow of a viscous incompressible electrically conducting fluid over an exponentially stretching sheet with viscous dissipation, radiation and internal heat generation. The governing partial differential equations are transformed to a set of ordinary differential equations using similarity transformation and are solved by applying Rungekutta fourth order method along with the shooting technique. The effects of Magnetic parameter, radiation parameter, Prandtl number, Eckerts number and heat source parameter on the heat transfer characteristics are obtained and discussed in detail with graphical representation.

KEY WORDS: MHD, Viscous dissipation, Heat transfer, Radiation, Exponentially stretching sheet, Boundary layer.

1. INTRODUCTION:

The study of two dimensional boundary layer flows on a continuous stretching sheet has acquired considerable attention due to its numerous applications in industrial, manufacturing processes such as hot rolling, glass fiber and paper production, drawing of plastic films, plastic sheets, metal and polymer processing industries.Sakiadis [1] was the first person who studied the boundary layer flow over a stretching sheet whose velocity varies linearly with the distance from a fixed point in the sheet. In 1970 Crane [2] extended this idea for the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point.Bidin B, and Nazar R (2009) [3] studied the numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation.

E.I. Aziz, M.A. (2009) [4] investigated viscous dissipation effect on mixed convection flow of a micro polar fluid over an exponentially stretching sheet. E. Magyari and B. Keller, [5] studied the heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. A.Ishak [6] studied the MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. Many other problems on exponentially stretching surface were discussed by Raptis et al.[11], Pardha et al. [12] andSajid and Hayat [13]. Jat and Chaudhary[14], studied the MHD boundary layer flow near the stagnation point of a stretching sheet.Recently R.N. Jat and GopiChand [18], investigated MHD flow and heat transfer over an exponentially stretching sheet with viscous dissipation and radiation.

The study of magneto hydrodynamic has important applications and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction in the fluid in the direction perpendicular to the motion and to the magnetic field.

It is now well accepted fact that the terms magnetohydrodynamics (MHD) thermal radiation and heat generation extensively appear in various engineering processes. MHD is significant in the control of boundary layer flow and metallurgical processes. The thermal radiation and heat generation possessions may arise in high temperature ingredients processing operations. Ingredients may be intelligently designed therefore with judicious implementation of radiative heating to produce the desired characteristics. This recurrently occurs in agriculture, engineering, plasma studies and petroleum industries. The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and these concerned with dissociating fluids. Heat generation is also important in the content of exothermic or endothermic chemical reactions.

The present paper studies the effects of radiation, viscous dissipation and heat source parameteron MHD boundary layer flow over an exponentially stretching sheet.The governing coupled, nonlinear partial differential equations of the flow of heat and transfer problems are transferred into non-linear, coupled ordinarydifferential equations by using a similarity transformation. These coupled non-linear ordinary differential equations subject to the appropriate boundary conditions are solved numerically Rungekutta fourth order method along with the shooting technique.

2 MATHEMATICAL FORMULATION

Consider a steady two dimensional laminar flow of a viscous incompressible electrically conducting fluid over a continuous exponentially stretching surface.The x-axis is taken along the stretching surface in the direction of motion and y-axis is perpendicular to it. We consider that a uniform magnetic field of strength B_0 is applied normal to the stretching surface (fig-1). The magnetic Reynolds number is taken to be small and therefore the induced magnetic field is neglected. The surface is assumed to be highly elastic and is stretched in the x-direction with a velocity $U=U_0 e^{\frac{x}{L}}$. All the fluid properties are assumed to be constant throughout the motion. Under the usual boundary layer approximations, the governing boundary layer equations by considering the viscous dissipation, radiation and heat source/sink effects are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0{}^2}{\rho}u \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0{}^2u^2+Q\,(T-T_\infty) \tag{3}$$

Where $Q=Q_0e^{\frac{x}{L}}$

where u and v are the velocity in the x-and y-directions respective, ρ is the density of the fluid, μ is the dynamic viscosity, $v=\frac{\mu}{\rho}$ is the kinetic viscosity, Cp is the specific heat at constant pressure, k is thermal conductivity of the fluid under consideration qr is the Radiative heat flux, Tis the temperature.

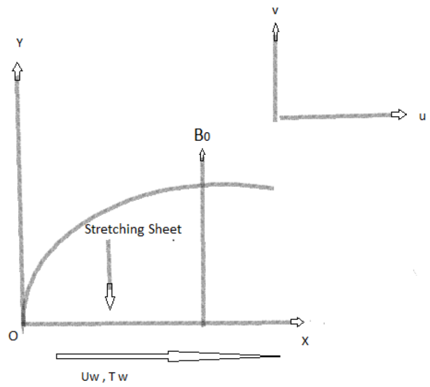


Fig.: Physical modal and Coordinate System

The boundary conditions are;

$$u=U_w=U_0\boldsymbol{e}^{\frac{x}{L}},\ v=0,\ T=T_\infty+T_0\boldsymbol{e}^{\frac{x}{2L}},\ \text{at } y=0$$

$$u\to 0,\ T\to 0 \text{ as } y\to\infty \tag{4}$$

where $\mathbf{U}_0$, $\mathbf{T_0}$ and $\mathbf{L_0}$are the reference velocity, temperature and length respectively. The radiative heat flux qr is simplified by using Rosseland [17] approximation as ;

$$q_r\cong\frac{-4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where$\boldsymbol{\sigma}$ is the Stefan –Boltzmann constant and $\boldsymbol{k}^*$ is the mean absorption coefficient.

This approximation is valid at points optically far from the boundary surface and it is good for intensive absorption, which is for an optically think boundary layer. It is assumed that the temperature difference with in the flow such that the term T may be expressed as a linear function of temperature. Hence, expanding T by Taylor series about T and neglecting higher-order term gives;

$$T^4\cong 4T_\infty{}^3T-3T_\infty{}^4 \tag{6}$$

Substituting (5) and (6) in eq^n (3) gives

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k+\frac{16\sigma T_\infty{}^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0{}^2u^2+Q\,(T-T_\infty) \tag{7}$$

ANALYSIS

The equation of continuity (1) is identify satisfied if we choose the stream function such that

$$u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x} \tag{8}$$

The momentum and energy equation can be transformed in to the corresponding ordinary differential equation by introducing the following similarity transformation;

$$\boldsymbol{\psi(x,y) = \sqrt{2vU_0L}e^{\frac{x}{2L}}f(\eta)}, \quad \boldsymbol{e^{\frac{x}{2L}}\theta(\eta) = \frac{T-T_\infty}{T_0}} \tag{9}$$

where

$$\eta = \left[\frac{U_0}{2vL}\right]^{\frac{1}{2}} ye^{\frac{x}{2L}}$$

Then, then the momentum and energy equation (2) and (7) are transformed to ;

$$\boldsymbol{f''' + ff'' - 2(f')^2 - Mf' = 0} \tag{10}$$

$$\boldsymbol{[1 + 4R/3]\theta'' + Pr(f\theta' - f'\theta + Ec(f'')^2 + MEc(f')^2 + \gamma\theta) = 0} \tag{11}$$

The corresponding boundaryconditions are ;

$f(0) = 0, f'(0) = 1, \theta(0) = 1$
$at\ \eta = 0$.
$f'(\eta) \to 0, \theta(\eta) \to 0\ as$
$\boldsymbol{\eta} \to \infty$(12)

Where prime (‘) denote the differentiation with respect to n and dimensionless parameters are;

$M = \frac{2\sigma{B_0}^2L}{\rho U_0 e^{\frac{x}{L}}}$(Magneticparameter),

$Ec = \frac{{U_0}^2}{c_pT_0}$(Eckerts Number),

$\gamma = \frac{2Q_0\mathrm{L}}{\rho c_pU_0}$(Heat source parameter),

$Pr = \frac{\mu cp}{k}$(Prandtl number), and

R = $\frac{4\sigma{T_\infty}^3}{kk^*}$(Radiation Parameter). (13)

The physical quantities of interest are the skin-friction coefficient Cf and the heat transfer rates i.e. is theNusselt number Nu are;

$$C_f = \frac{2\tau_w}{\rho{U_w}^2} \tag{14}$$

And the local Nusselt number:

$$Nu = \frac{xq_w}{(T_w - T_\infty)} \tag{15}$$

Where the surface shear stress τ_w and the surface heat flux $q_w$are

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$q_w = -\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{16}$$

With μ being the dynamic viscosity. Using non dimensional variable (9) in (16) we obtain:

$$\frac{1}{2}C_f\sqrt{Re} = f''(0),$$

$$\frac{Nu}{\sqrt{Re}} = -\theta'(0) \tag{17}$$

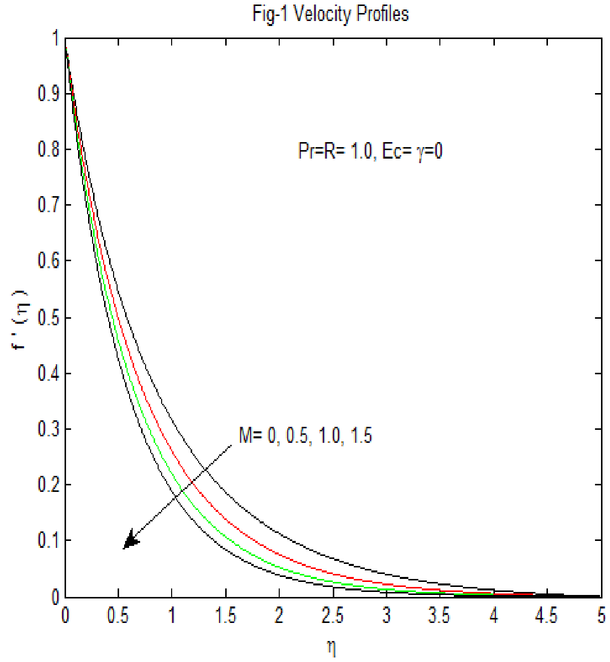
Where $Re = \frac{U_0L}{v}$is the local Reynold number.

RESULTS AND DISCUSSION

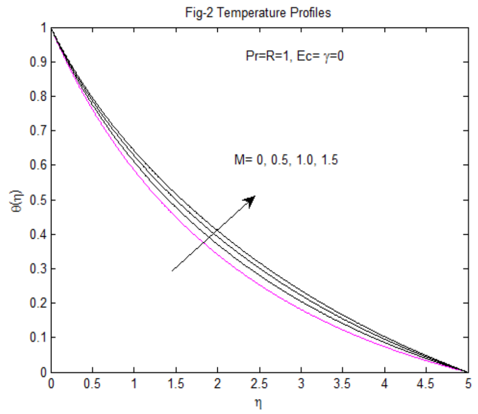
The set of nonlinear ordinary differential equation (10) and (11) with boundary conditions (12) were solved numerically using Rungekutta fourth order method along with the shooting technique. This method has been successfully used to solve various boundary layer problems along with the concept of similarity solution.Comparison with the existing results from the literature shows a favorable agreement as prescribed in Table 1.

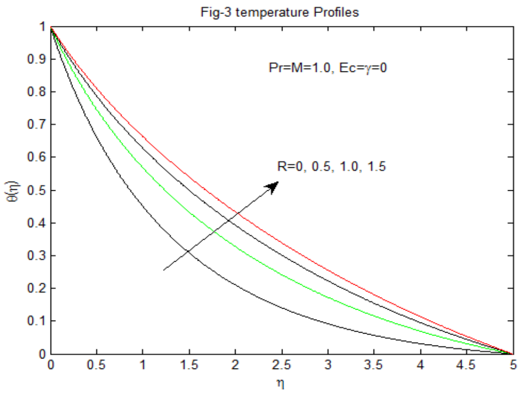
To access the effects of the various parameters on the flow and heat transfer characteristics, the numerical results are presented in Figs.1-11 and in Table 2.The velocity profiles for various values of the Magnetic parameter M presented in Fig-1 show that the rate of transport is considerably reduced with the increase of M. It is observed that velocity decreases with the increasing values of M. This is because the increasing value of M tends to the increasing of Lorentz force, which produces more resistance to the transport phenomena.

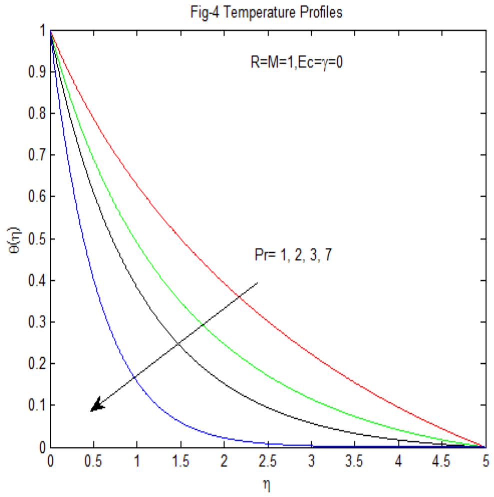
Fig-1 Velocity Profiles



The temperature profiles for various values of M, R, Pr keeping Ec and heat source parameter γas zeroes and other parameters are fixed to unity are presented in Figs.2, 3, and 4, respectively.It is observed fromthe figures that theboundaryconditions are satisfied asymptotically in all cases, which supports the accuracy of the results obtained. From these Figs it is evident that the thermal boundary layer thickness increases as M and R increase but opposite trends are observed for increasing values of Pr.







The velocity profiles for various values of the Magnetic parameter M presented in Fig-5

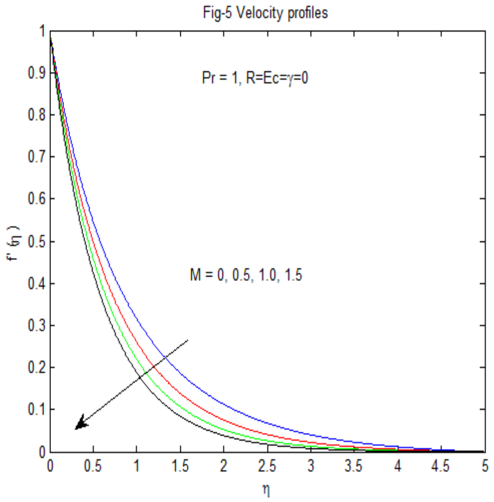
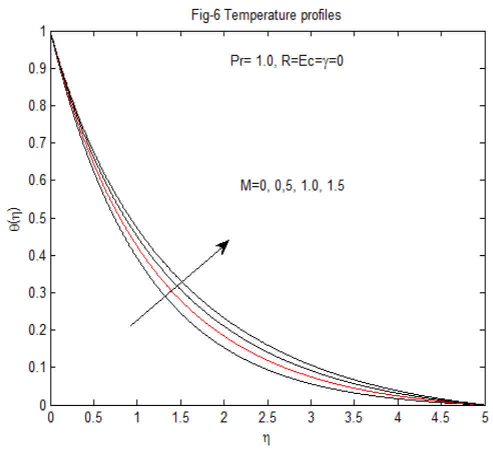


TABLE 1 Values of $-\theta'(0)$ for different values of R, M and Pr

R	M	Pr	Magyari& Keller(1999)	Bidin&Nazar (2009)	A Ishak (2011)	Present
0	0	1	0.954782	0.9548	0.9548	0.954809
		2		1.4714	1.4715	1.471454
		3	1.869075	1.8691	1.8691	1.869073
		5	2.500135		2.5001	2.500148
		10	3.660379		3.6604	3.660465
	1	1			0.8611	0.861507
1	0			0.5315	0.5312	0.535300
	1				0.4505	0.450513

The temperature profiles for various values of M, R, Pr, Ec andγ are presented in Figures 6 to 11. All the figures show that the increasing value of any parameter except Pr, result is increase the thermal boundary layer thickness, where as increase in Pr is to decrease the thermal boundary layer thickness.



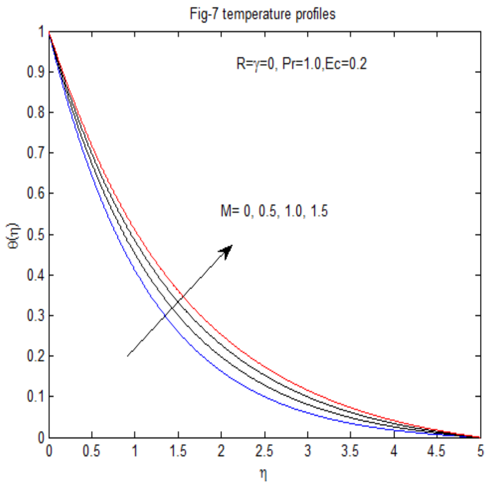


Fig-8 Temperature profiles

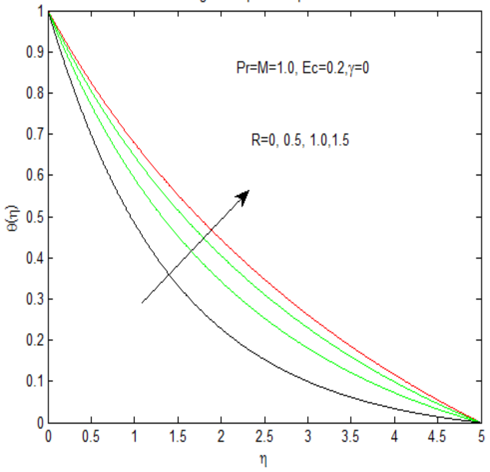


Fig-9 Temperature profiles

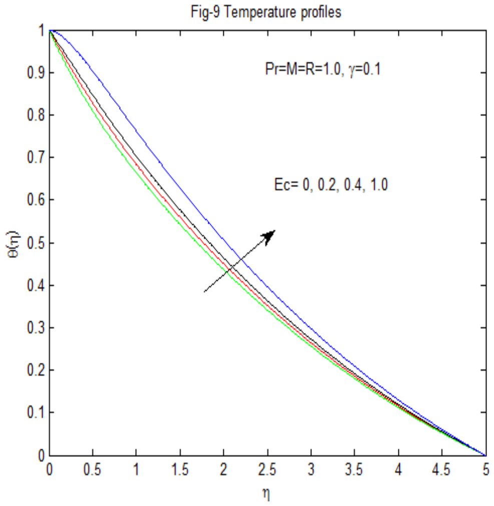


Fig-10 Temperature Profiles

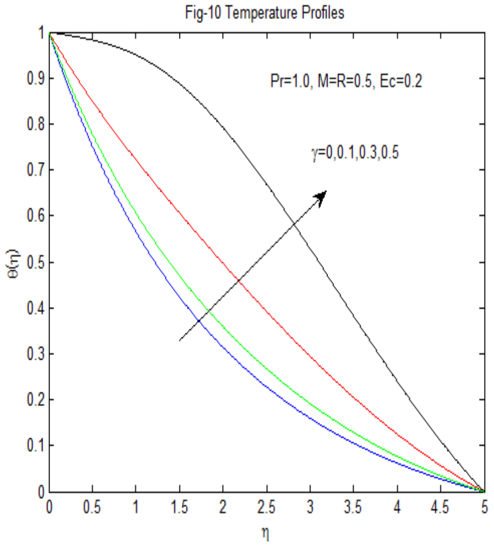
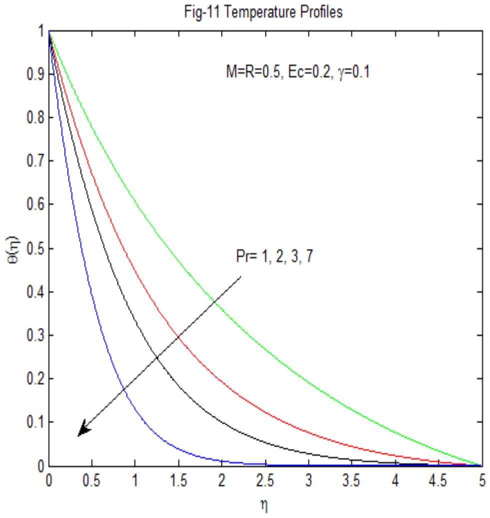


Fig-11 Temperature Profiles



The computationsweredone by a program which uses a symbolic and computation computer language matlab. The shear stress which is proportion to $f''(0)$ and the rate of heat transfer which is proportional to $-\theta'(0)$ are tabulated in Table 2.0 for different values of parameter. It is observed from the table that the shear stress decreases and heat transfer rate increases as magnetic parameter increases. Also the Nusselt number decreasing with theincreasing value of Ec for a given Pr, where as it is increases for increasing value of Pr for a given value of Ec.

TABLE 2.0 Numerical values of Skin friction Coefficient and Nusselt number For M = 0.0, 0.5, 1.0

Pr	R	Ec	γ	$-f''(0)$	$-\Theta'(0)$	$-f''(0)$	$-\Theta'(0)$	$-f''(0)$	$-\Theta'(0)$
1	0	0.1	0.1	1.281813	0.831427	1.466442	0.737341	1.629174	0.652363
7				1.281813	2.657403	1.466442	2.474973	1.629174	2.312110
1	0.5			1.281813	0.570822	1.466442	0.486638	1.629174	0.416244
		0.2		1.281813	0.540318	1.466442	0.439843	1.629174	0.657294
			0.2	1.281813	0.444441	1.466442	0.256023	1.629174	0.016067
			0.0	1.281813	0.647959	1.466442	0.585195	1.629174	0.534891
			-0.1	1.281813	0.708556	1.466442	0.655312	1.629174	0.612865
			-0.2	1.281813	0.760585	1.466442	0.713006	1.629174	0.675002

CONCLUSIONS

The effects of radiation parameter Magnetic parameter M, Pr, Ec and heat source parameter on the steady MHD boundary layer flow over an exponentially stretching sheet were investigated. The numerical results obtained agreed very well with previously reported cases available in the literature.

Some of the interesting conclusions are as follows:

(i) It is observed that the surface shear stress decreases with the magnetic parameter M.

(ii) The Nusselt number decreases with parameters Ec or R and for a given Pr.

(iii) The Nusselt number increases for increasing value of Pr for a given value of Ec.

(iv) The Nusselt number decreases with increasing values of heat source parameter $(\gamma > 0)$ and increases with the heat sink parameter $(\gamma < 0)$.

(v) The thermal boundary layer thickness increases as M and R increase but opposite trends are observed for increasing values of Pr.

(vi) The velocity decreases with the increasing values of M.

(vii) The increasing value of any parameter except Pr, result is increase the thermal boundary layer thickness, where as increase in Pr is to decrease the thermal boundary layer thickness.

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