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Unsteady Flow Of Visco-Elastic Liquid In The Boundary Layer Around A Circular Cylinder Oscillating Harmonically

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ABSTRACT:

In this paper we have studied the unsteady flow of a Visco-elastic liquid in the boundary layer around a body of revoluation in a circular cylinder when the body oscillates harmonically on the liquid at rest. We conclude that the dividing stream lines move away from the wall of the cylinder. The thickness of the inner vertex system increases and the intensity of the secondary flow is greater near the solid boundary.

Key Words: Unsteady flow, Boundary Layer, Stream function, Circular Cylinder

1. INTRODUCTION:

In the recent past, significant theoretical and experimental investigations have been performed on simple geometries to understand the role of elasticity on the flow of fluids. Again in the past few decades due to number of applications in industrial manufacturing process, the problem of boundary layer flow has attracted considerable attention of researchers. Examples of such technological process are hot rolling, wire drawing, glass-fiber and paper production. Some of the most notable of these are studied by Sakiadis (1961), Tsou et al. (1967), and Crane, L.J. (1970). Here we see that the growth of unsteady boundary layer where the body oscillates harmonically with the time in a liquid at rest is of great physical importance. Initially Schlichting (1932) obtained the solution of the two dimensional non-steady boundary layer equations for a viscous liquid where the free stream oscillates harmonically with time. The same type of flow of a fluid between two concentric cylinders which could be entirely in response to the fluctuations in the velocity of either inner or outer cylinder has been considered by Uchida (1956). Chang (1974) and Schowalter (1975) considered the flow of visco-elastic liquid near an oscillating cylinder. Boundary layer flows in a visco-elastic liquid of oscillating cylinder has been considered by Chang (1977). Rath and Jena (1979) studied the flow of a viscous fluid generated in response to fluctuation in the axial velocity of the outer cylinder. Biswal, Mishra and Pratihari (1985) studied the above problem in case of visco-elastic liquid. Flow and heat transfer in a visco-elastic fluid over a stretching sheet is studied by Dhanpat and Gupta (1989). Ahmad, Patel and Siddappa (1990) studied the visco-elastic boundary layer flow past a stretching plate and heat transfer. The same problem was studied with suction and heat transfer by Ahmad et al. (1991).

Visco-elastic boundary layer flow past a stretching plate with suction heat transfer with variable conductivity is studied by Ahmad and Marwah (1999). The same type of flow is applicable in bio fluid dynamics. Here the flow of non-Newtonian visco-elastic fluids in a channel over a stretching sheet has been attracted attention of researchers because of its many applications in engineering and industry. Datti et al. (2004) studied above type of fluids and got interesting results. Misha et al. (2011) studied Hydro magnetic flow and heat transfer of a second grade visco-elastic fluid in a channel with oscillatory stretching walls. This problem is also applied to dynamics of blood flow. In 2012 Sahoo et al. studied the flow and heat transfer of visco-elastic liquids between walls having periodic deformation. Prior to this Andesson et al. (2001) considered the basic flow solution for shear-thickening and shear-thinning power law fluids. But the others overlooked the importance of matching this boundary layer solution to an external flow. Denier and Hewitt (2004) addressed this problem and presented the corrected solution for both cases. Few years back, Ahmadpour and Sadeghy (2013) addressed the problem of the flow due to a rotating disk when one considers Bingham plastic fluids. The authors have claimed to have found an exact solution to the problem and are only able to present numerical solutions for specific values of Reynolds number and dimensionless radius of the disk. Griffiths (2015) has considered the boundary layer flow due to a rotating disk for a number of generalized Newtonian fluid models. In this paper, the flow of a visco-elastic liquid in the boundary layer around a body of revolution in particular reference to a circular cylinder has been studied when the body oscillates harmonically on the liquid at rest. In a way, this problem is an extension of the work done by Griffiths (2015).

2. BOUNDARY LAYER EQUATIONS

Here we need to solve the equations of boundary layer over a body of revoluation when the stream is parallel to its axis. In Cartesian frame of reference, we can write the modified Navier-Stokes equations in the form

$$\rho\left[\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}\right]=-\frac{\partial p}{\partial x_i}+\mu\frac{\partial^2 u_i}{\partial x_k\partial x_k}-k\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 u_i}{\partial x_k\partial x_k}\right)+u_m\frac{\partial^3 u_i}{\partial x_m\partial x_k\partial x_k}-\frac{\partial u_i}{\partial x_m}.\frac{\partial^2 u_i}{\partial x_k\partial x_k}-2\frac{\partial^2 u_i}{\partial x_k\partial x_k}\frac{\partial u_m}{\partial x_k}\right] \quad (1)$$

We now make the usual boundary layer assumptions of the viscous flow theory. The same assumptions are also applicable in visco-elastic case with in the boundary layer, u, $\frac{\partial u}{\partial x}$, $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial p}{\partial x}$ are assumed to be O(1), and y to be O(δ) where δ is the thickness of the boundary layer near a solid boundary $y=0$. From the equation of continuity, it can be easily seen that $v=\mathrm{O}(\delta)$. In order that the viscous, visco-elastic and inertia terms in the equation of motion shall be of same order of magnitude, it is necessary that

$$v=\frac{\mu}{\rho}=O(\delta^2), k^*=\frac{k}{\rho}=O(\delta^2)$$

Under the above conditions, the boundary layer equations are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-k^*\left[\frac{\partial^3 u}{\partial t\partial y^2}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}.\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}.\frac{\partial^2 u}{\partial x\partial y}\right] \quad (2)$$

$$\frac{\partial p}{\partial y}=O(\delta). \quad (3)$$

3. **SOLUTION OF BOUNDARY LAYER EQUATIONS**

Here we solve the boundary layer equations (2) and (3) subject to the boundary conditions

$$\left.\begin{array}{l} y=0, u=v=0 \\ y\to\infty, u\to U(x,t) \end{array}\right\} \quad (4)$$

Where $U(x,t)$ denotes the partial flow about the body of revolution. The pressure in the in viscid flow is given by

$$-\frac{\partial p}{\partial x}=\rho\left[\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}\right] \quad (5)$$

Introducing the expression above for $\frac{\partial p}{\partial x}$ into equation (2) we have

$$\begin{aligned}&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\\ &k^*\left[\frac{\partial^3 u}{\partial t\partial y^2}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}.\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}.\frac{\partial^2 u}{\partial x\partial y}\right]\end{aligned} \quad (6)$$

In order to solve equation (6) we apply the general approximation as follows

$$\left.\begin{array}{l} u(x,y,t)=u_0(x,y,t)+u_1(x,y,t) \\ v(x,y,t)=v_0(x,y,t)+v_1(x,y,t) \end{array}\right\}$$

Introducing the above in equation (6), we get

$$\frac{\partial u_0}{\partial t} - v\frac{\partial^2 u_0}{\partial y^2} + k^*\frac{\partial^3 u_0}{\partial t \partial y^2} = \frac{\partial U}{\partial t} \tag{7}$$

and
$$\frac{\partial u_1}{\partial t} - v\frac{\partial^2 u_1}{\partial y^2} + k^*\frac{\partial^3 u_1}{\partial t \partial y^2} = U\frac{\partial U}{\partial x} - u_0\frac{\partial u_0}{\partial x} - v_0\frac{\partial u_0}{\partial y} - k^*\left[\frac{\partial u_0}{\partial y}\frac{\partial^2 u_0}{\partial y^2} + u_0\frac{\partial^3 u_0}{\partial x \partial y^2} + \frac{\partial u_0}{\partial y}\frac{\partial^2 v_0}{\partial y^2} + v_0\frac{\partial^3 u_0}{\partial y^3}\right] \tag{8}$$

Boundary conditions (4) give

$$\left.\begin{array}{l} y = 0, u_0 = 0 = u_1 \\ y \to \infty, u_0 = U(x,t), u_1 = 0 \end{array}\right\} \tag{9}$$

Now we take the potential flow about the body of revolution to be

$$U(x,t) = U_0(x)\cos(nt) \tag{10}$$

In complex notation, we take it as

$$U(x,t) = U_0(x)\exp(\text{int}) \tag{11}$$

Here only the real parts of the complex quantities in problem have physical meaning attached to them. We introduce the dimensionless quantity

$$\eta = y(n/\upsilon)^{1/2} \tag{12}$$

And assume that the first approximation to the stream function ψ_0 is of the form

$$\psi_0(x,y,t) = -(\upsilon/n)^{1/2}U_0(x).r(x)F(\eta)e^{\text{int}} \tag{13}$$

Hence in view of equation of continuity, we have

$$ru_0 = U_0 rF'(\eta)e^{\text{int}} \tag{14}$$

And $rv_0 = -(\upsilon/n)^{1/2}\left[r\frac{dU_0}{dx} + U_0\frac{dr}{dx}\right]F(\eta)e^{\text{int}}$ (15)

From equation (7) and (14), we get

$$iF' - (1 - i\alpha)F''' = i \tag{16}$$

Where

$\alpha = k^* n /$

It is clear that α is small enough so that its second and higher powers can be neglected.

The differential equation (16) is to be solved under the boundary conditions

$$\left.\begin{array}{l} \eta = 0 : F(\eta) = F'(\eta) = 0 \\ \eta \to \infty : F'(\eta) = 1 \end{array}\right\} \tag{17}$$

Solving (16) with the conditions (17), we get

$$F'(\eta) = 1 - e^{-p\eta} \tag{18}$$

Where $p = \dfrac{[\beta(\alpha)+\alpha]^{1/2} + i[\beta(\alpha)-\alpha]^{1/2}}{\sqrt{2}\beta(\alpha)} = p_1 + ip_2$

And $\beta(\alpha) = \sqrt{1+\alpha^2}$ (19)

From (14) and (18), we get

$$u_0(x,y,t)=U_0(x)\left[1-e^{-p\eta}\right]e^{\text{int}} \tag{20}$$

Changing to real notation, we obtain

$$u_0(x,y,t)=U_0(x)\left[\cos(nt)-e^{-p_1\eta}\cos(p_2\eta-nt)\right]$$

Here the viscous solution is obtained a particular case when $\alpha=0$.

We express the second approximation to the stream function ψ_1 in the form

$$\psi_1(x,y,t)=(\upsilon/n)^{1/2}\left[\frac{rU_0}{n}\frac{dU_0}{dx}\left\{F_1(\eta)e^{2\text{int}}+F_2(\eta)\right\}+\frac{U_0^2}{n}\frac{dr}{dx}\left\{F_3(\eta)e^{2\text{int}}+F_4(\eta)\right\}\right] \tag{21}$$

Hence $u_1(x,y,t)=\dfrac{U_0}{n}\dfrac{dU_0}{dx}\left\{F_1'(\eta)e^{2\text{int}}+F_2'(\eta)\right\}+\dfrac{U_0^2}{rn}\dfrac{dr}{dx}\left\{F_3'(\eta)e^{2\text{int}}+F_4'(\eta)\right\}$ (22)

Real parts of equations (11), (14), (15), and (22) can be written in the form

$$U(x,t)=(U_0/2)\left[e^{\text{int}}+e^{-\text{int}}\right] \tag{23}$$

$$u_0=(U_0/2)\left[Fe^{\text{int}}+\overline{F}e^{-\text{int}}\right] \tag{24}$$

$$v_0=-(\upsilon/n)^{1/2}\left[\frac{dU_0}{dx}+\frac{U_0}{r}\frac{dr}{dx}\right]\left[Fe^{\text{int}}+\overline{F}e^{-\text{int}}\right] \tag{25}$$

$$u_1=\frac{U_0}{n}\frac{dU_0}{dx}\left[\frac{1}{2}\left\{F_1'e^{2\text{int}}+\overline{F}_1'e^{-2\text{int}}\right\}+F_2'\right]+\frac{U_0^2}{nr}\frac{dr}{dx}\left[\frac{1}{2}\left\{F_3'e^{2\text{int}}+\overline{F}_3'e^{-2\text{int}}\right\}+F_4'\right] \tag{26}$$

Where the bar over a symbol denotes the corresponding conjugate complex quantity.

Substituting (23)-(26) in (8) and equating the coefficients of like terms, we get the set of differential equations.

$$2iF_1'-(1-2i\alpha)F_1'''=\frac{1}{2}\left[1-F'^2+FF''\right]-\alpha\left[F'F'''-\frac{1}{2}(F''^2+FF^{iv})\right] \tag{27}$$

$$-F_2'''=\frac{1}{2}-\frac{1}{2}F'\overline{F}'+\frac{1}{4}(F\overline{F}''+\overline{F}F'')-\alpha\left[\frac{1}{2}F'\overline{F}'''+\overline{F}'F'''-\frac{1}{2}F''\overline{F}''-\frac{1}{4}(F\overline{F}^{iv}+\overline{F}F^{iv})\right] \tag{28}$$

$$2iF_3'-(1-2i\alpha)F_3'''=\frac{1}{2}FF''-\frac{1}{2}\alpha(F''^2+FF^{iv}) \tag{29}$$

$$-F_4'''=\frac{1}{4}(F\overline{F}''+\overline{F}F'')-\frac{1}{2}\alpha\left[F''\overline{F}''+\frac{1}{2}(F\overline{F}^{iv}+\overline{F}F^{iv})\right] \tag{30}$$

The corresponding boundary conditions are as follows

$$\left.\begin{aligned}&\eta=0: F_1=F_1'=0, F_2=F_2'=0\\&F_3=F_3'=0, F_4=F_4'=0\end{aligned}\right\} \tag{31}$$

$$\left.\begin{aligned}&\eta\to\infty: F_1'\to 0, F_3'\to 0\\&F_2'=\mathit{finite}, F_4'=\mathit{finite}\end{aligned}\right\} \tag{32}$$

Solving the equations (27)-(30) subject to the boundary conditions (31) and (32), we get

$$F_1' = \left(\frac{k}{p^2 - m^2}\right)\left[\left\{1 + 2p^2/(p^2 - m)\right\}\left\{e^{-(n_1 + in_2)\eta} - e^{-p\eta}\right\} - \eta p e^{-p\eta}\right] \tag{33}$$

$$F_2' = M_1 e^{-2p_1\eta} + M_2 e^{-p_1\eta}\cos(p_2\eta) + M_3 e^{-p_1\eta}\sin(p_2\eta) + M_4\eta e^{-p_1\eta}\cos(p_2\eta) + M_5\eta e^{-p_1\eta}\sin(p_2\eta) - M_6 \tag{34}$$

$$F_3' = \left[\frac{(1-\alpha p^2)(p^2+m)}{(p^2-m)^2} + \frac{1-2\alpha p^2}{4p^2 - m}\right]\left(\frac{e^{-(n_1 - in_2)\eta}}{2(1-2i\alpha)}\right) - \frac{(1-\alpha p^2)e^{p\eta}}{2(1-2i\alpha)(p^2-m)}\left[p\eta + \frac{p^2+m}{p^2-m}\right] - \frac{(1-2\alpha p^2)e^{-2p\eta}}{2(1-2i\alpha)(4p^2-m)} \tag{35}$$

$$F_4' = N_1 e^{-2p_1\eta} + N_2 e^{-p_1\eta}\cos(p_2\eta) + N_3 e^{-p_1\eta}\sin(p_2\eta) + N_4\eta e^{-p_1\eta}\cos(p_2\eta) + N_5\eta e^{-p_1\eta}\sin(p_2\eta) - N_6 \tag{36}$$

Where $k = \dfrac{1+\alpha p^2}{2(1-2i\alpha)}, m = m_1 + im_2 = \dfrac{-4\alpha + 2i}{1+4\alpha^2}$

$$n_1 + in_2 = \frac{\{\beta(2\alpha) + 2\alpha\}^{1/2} + i\{\beta(2\alpha) - 2\alpha\}^{1/2}}{\beta(2\alpha)}$$

$$M_1 = \frac{p_2^2 M_7}{4p_1^2}$$

$$M_2 = 2p_1p_2M_8 - (p_1^2 - p_2^2)M_9 + \frac{2p_1(3p_2^2 - p_1^2)M_{10} - 2p_2(3p_1^2 - p_2^2)M_{11}}{p_1^4(p_1^2 + 3p_2^2) + p_2^4(p_2^2 + 3p_1^2)}$$

$$M_3 = 2p_1p_2M_9 + (p_1^2 - p_2^2)M_8 + \frac{2p_2(3p_1^2 - p_2^2)M_{10} + 2p_1(3p_2^2 - p_1^2)M_{11}}{p_1^4(p_1^2 + 3p_2^2) + p_2^4(p_2^2 + 3p_1^2)}$$

$$M_4 = -\frac{2p_1p_2M_{11} + (p_1^2 - p_2^2)M_{10}}{(p_1^2 + p_2^2)^2}$$

$$M_5 = -\frac{2p_1p_2M_{10} - (p_1^2 - p_2^2)M_{11}}{(p_1^2 + p_2^2)^2}$$

$$M_6 = M_1 + M_2$$

$$M_7 = \frac{1 + 2\alpha(p_1^2 - p_2^2)}{p_1^2 + p_2^2}$$

$$M_8 = \frac{p_1p_2(1 - 4\alpha p_2^2)}{(p_1^2 + p_2^2)^3}$$

$$M_9 = \frac{(p_1^2 + 3p_2^2) + \alpha(p_1^4 + 6p_1^2p_2^2 - 3p_2^4)}{2(p_1^2 + p_2^2)^3}$$

$$M_{10} = \frac{1}{2}p_1\left[1 + \alpha(p_1^2 - 3p_2^2)\right]$$

$$M_{11} = \frac{1}{2} p_2 \left[1 + \alpha(3p_1^2 - p_2^2)\right]$$

$$N_1 = \frac{-(p_1^2 - p_2^2)M_7}{8p_1^2}$$

$$N_2 = \frac{(p_1^2 - p_2^2)N_7 + 2p_1 p_2 N_8}{(p_1^2 + p_2^2)^2} + 2p_1(3p_2^2 - p_1^2)M_{10}$$

$$+ \frac{2p_1(3p_2^2 - p_1^2)M_{10} + 2p_2(p_2^2 - 3p_1^2)M_{11}}{p_1^4(p_1^2 + 3p_2^2) + p_2^4(p_2^2 + 3p_1^2)}$$

$$N_3 = \frac{(p_1^2 - p_2^2)N_8 - 2p_1 p_2 N_7}{(p_1^2 + p_2^2)^2} + \frac{2p_2(3p_1^2 - p_2^2)M_{10} + 2p_1(3p_2^2 - p_1^2)M_{11}}{p_1^4(p_1^2 + 3p_2^2) + p_2^4(p_2^2 + 3p_1^2)}$$

$$N_4 = M_4, N_5 = M_5, N_6 = N_1 + N_2$$

$$N_7 = \frac{(p_1^2 - p_2^2) + \alpha(p_1^4 - 6p_1 p_2 + p_2^4)}{2(p_1^2 + p_2^2)^3}, N_8 = p_1 p_2 M_7$$

It is observed that the second approximation contains a steady state term which does not vanish outside the boundary layer that is at a large distance from the body. The velocity field, correct to the second approximation can be written as

$$u = \mathrm{Re}\left[U_0 F' e^{\mathrm{int}} + \frac{U_0}{n}\frac{dU_0}{dx}\left\{F_1' e^{2\mathrm{int}} + F_2'\right\} + \frac{U_0^2}{rn}\frac{dr}{dx}\left\{F_3' e^{2\mathrm{int}} + F_2'\right\}\right] \tag{37}$$

Taking r(x)=Constant, the body of revolution becomes a circular cylinder. In that case the circular cylinder oscillates in a stream of velocity U_∞.

The potential flow in this case is given by

$$U(x,t) = U_\infty \sin(\pi x / R)e^{\mathrm{int}} \tag{38}$$

Where R/π is the radius of the cylinder. In this case equation (37) is reduced to

$$u = \mathrm{Re}\left[U_0 F' e^{\mathrm{int}} + \frac{U_0}{n}\frac{dU_0}{dx}\left\{F_1' e^{2\mathrm{int}} + F_2'\right\}\right] \tag{39}$$

In order to consider the steady streaming in the potential flow, let us consider the average velocity component $\bar{u}$ which is defined as

$$\bar{u} = \frac{u}{2\pi}\int_0^{2\pi} u\,dt \tag{40}$$

From (39) and (40), we get

$$\bar{u} = \frac{U_0}{n}\frac{dU_0}{dx} F_2'(\eta) \tag{41}$$

Whereas from (11) and (38), we get

$$U_0(x) = U_\infty \sin(\pi x / R) \tag{42}$$

The stream function corresponding to (41) is given by

$$\bar{\psi} = \sqrt{\upsilon / n}\, U_0 \frac{dU_0}{dx}\frac{1}{n} F_2(\eta) \tag{43}$$

From (42) and (41), we have

$\overline{\psi} = (U_\infty^2 / 2n)(\pi / R)\sqrt{\upsilon / n}\sin(2\pi x / R)F_2(\eta)$

Using the dimensionless parameter $x' = (x / R)$, the non-dimensional stream function will be given by

$$\psi = \frac{\overline{\psi}\, 2nR}{\pi\sqrt{\upsilon / n}U_\infty^2} = \sin(2\pi x')F_2(\eta) \qquad (44)$$

Integrating (35) subject to the condition in (31), we get

$$\begin{aligned} F_2(\eta) &= M_{12} - M_{13}\eta - M_{14}e^{-2p_1\eta} - M_{15}e^{-p_1\eta}\cos(p_2\eta) - M_{16}e^{-p_1\eta}\sin(p_2\eta) \\ &- M_{17}\eta e^{-p_1\eta}\cos(p_2\eta) - M_{18}\eta e^{-p_1\eta}\sin(p_2\eta) \end{aligned} \qquad (45)$$

Where $M_{12} = M_{14} + M_{15}, M_{13} = M_6, M_{14} = M_1 / 2p_1$

$$M_{15} = \frac{M_2 p_1 + M_3 p_2}{p_1^2 + p_2^2} + \frac{M_4(p_1^2 - p_2^2) + 2p_1 p_2 M_5}{(p_1^2 + p_2^2)^2}$$

$$M_{16} = \frac{M_3 p_1 - M_2 p_2}{p_1^2 + p_2^2} + \frac{M_5(p_1^2 - p_2^2) - 2p_1 p_2 M_4}{(p_1^2 + p_2^2)^2}$$

$$M_{17} = \frac{M_4 p_1 + M_5 p_2}{p_1^2 + p_2^2}$$

$$M_{18} = \frac{M_5 p_1 - M_4 p_2}{p_1^2 + p_2^2}$$

Neglecting the terms containing second and higher powers of α, it can also be seen that

$F_2'(\infty) = -\frac{3}{4} + 4\alpha$, Which in Newtonian case becomes $-\frac{3}{4}$ where $\alpha = 0$.

3. DISCOUSSION OF THE RESULTS

In this paper we have studied the unsteady flow of visco-elastic liquid in the boundary layer around a circular cylinder, when r(x)=constant, the solution obtained is not a uniformly valid solution except for certain value of the elastic parameter α which is $\alpha = 3/16$ (in the first order approximation). This is because the solution does not satisfy the boundary conditions at infinity due to the boundary layer approximation.

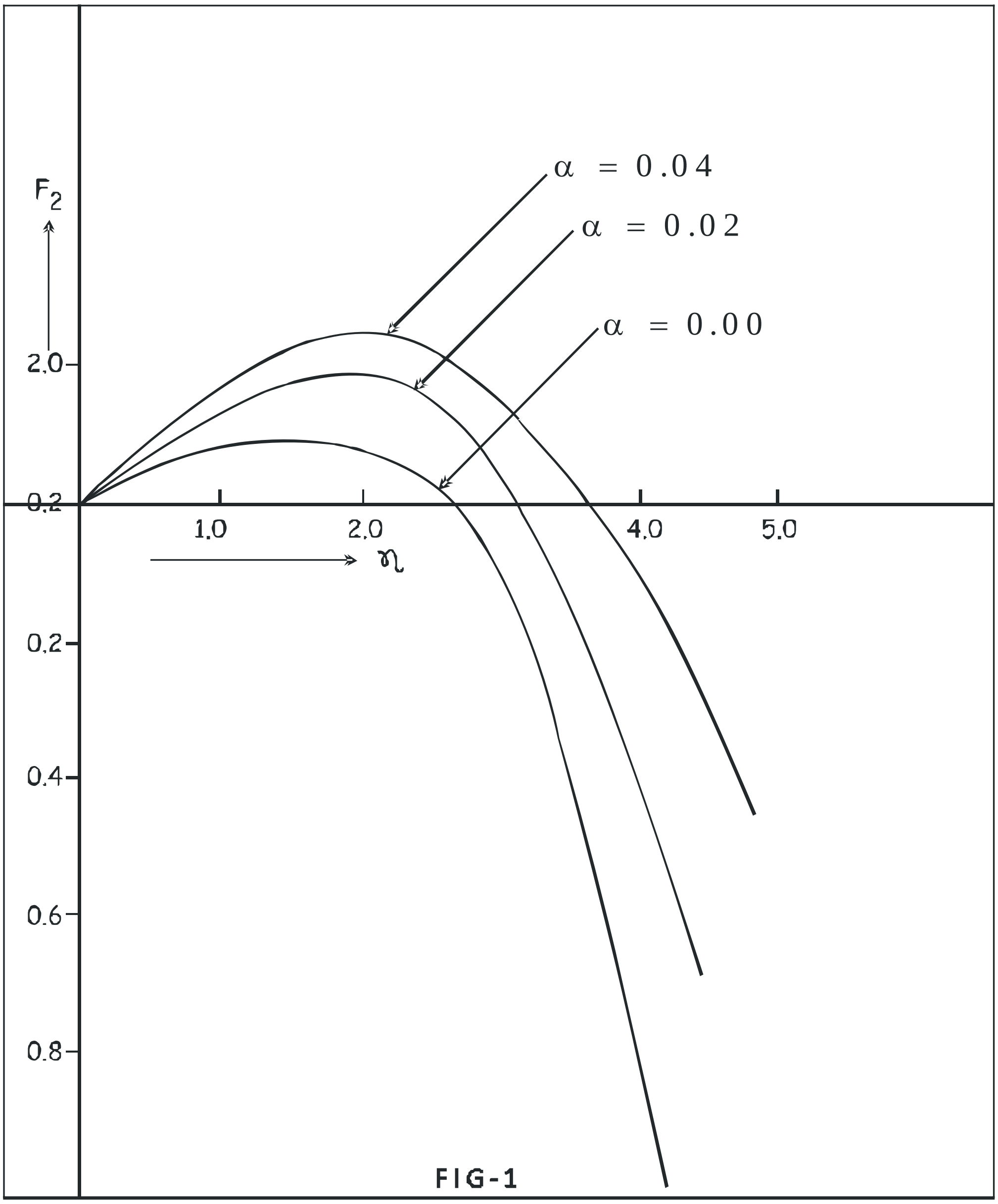
From the equation (44), it is seen that for $0 < x' < 0.5, \psi > 0 or < 0$ according as $F_2(\eta) > 0 or < 0$. Also when $F_2(\eta) = 0$, then $\psi = 0$. From the fig-1, it can be seen that $F_2(\eta) = 0$ at $\eta = 2.70, 3.14, 3.65\, for \alpha = 0.0, 0.2, 0.04$ respectively. We conclude that the dividing streamlines move away from the wall of the cylinder.

The effects of visco-elasticity of the liquid on F_2' and F_4' have been presented in fig-2. An examination of this figure shows that the effect of elasticity of the liquid is to increase both F_2' and F_4'. It means the thickness of the inner vertex system increases and the intensity of the secondary flow near solid boundary increases. In a thin liquid layer near the rigid body F_2' first increases and then decreases. It means that the secondary flow is greater near the solid boundary.

CAPTION OF THE DIAGRAMS

Figure 1: $F_2(\eta) = 0 \;\; at \;\; \eta = 2.70, 3.14, 3.65 \; for \alpha = 0.0, 0.2, \; and \; 0.04$

Figure 2: The effect of the elasticity of the liquid on F_2' and F_4' for different values of α.



Streamlines for different values of α

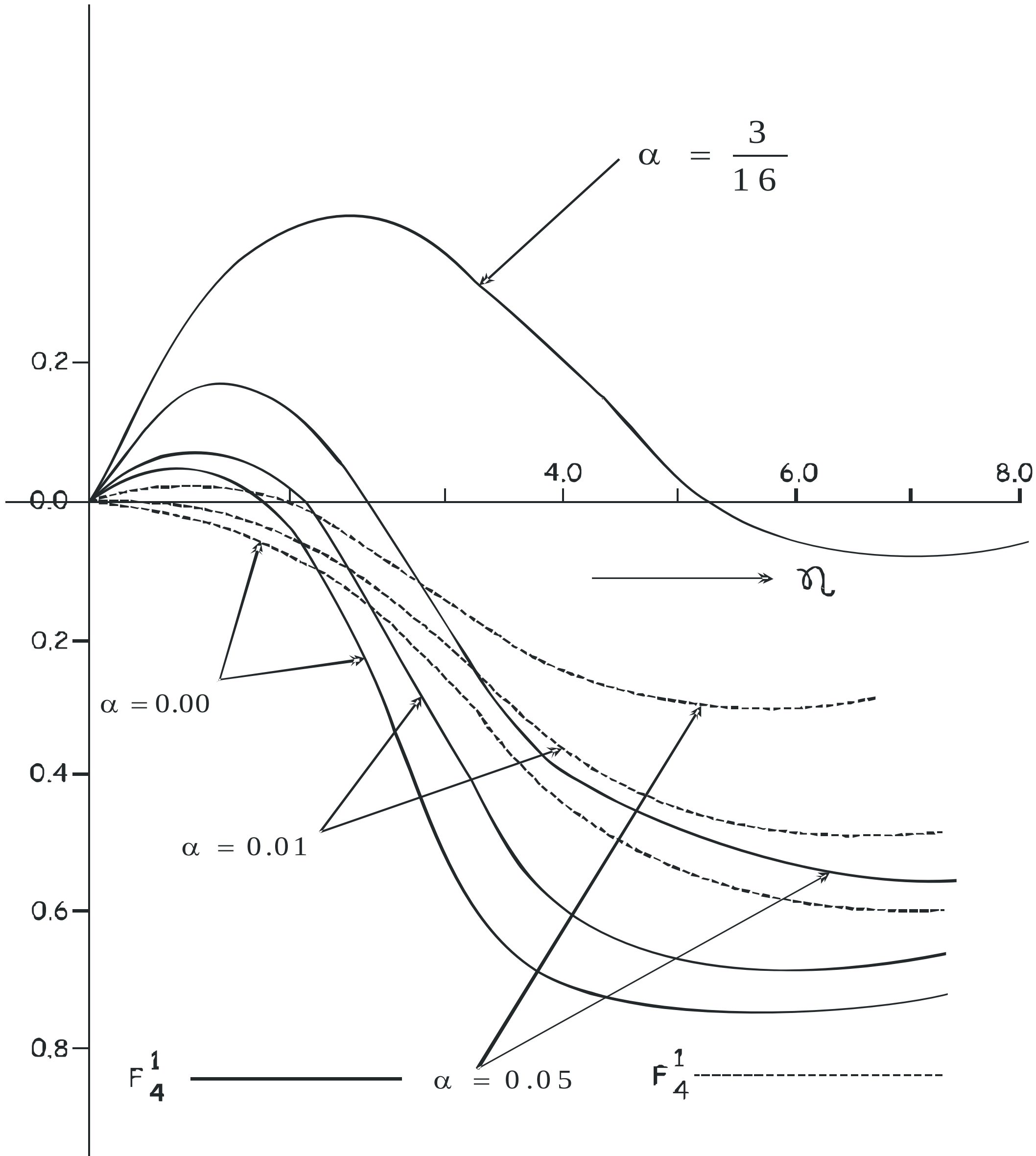


FIG-2

Streamlines for Different Values of α

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