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Influence of Chemical Reaction and Heat Generation on Heat and Mass Transfer due to a Rotating Heated Sphere

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ABSTRACT:

The paper presents numerical investigation of the effects of chemical reaction and heat generation on heat and mass transfer due to a rotating heated sphere in thermally and electrically conducting incompressible viscous binary fluid mixture in presence of magnetic field, viscous dissipation, baro-diffusion and thermal diffusion. The boundary layer equations governing the flow are first treated with expansions in ascending powers of $sin\theta$ and then solved numerically by using Matlab's boundary value problem solver bvp4c. The numerical values are tabulated and solutions are represented graphically. It has been found that increase in the heat generation parameter increases the temperature of the binary fluid mixture and decreases the concentration of the rarer and lighter component of the binary fluid mixture. Increase in the chemical reaction parameter increases the concentration of the rarer and lighter component of the binary fluid mixture at any point in the boundary layer.

Keywords: Rotating heated sphere, binary mixture, chemical reaction, heat generation, bvp4c.

INTRODUCTION

The problem of the flow due to a rotating sphere in a viscous fluid is a fascinating one which has numerous applications in engineering, industries, astrophysics and meteorology and has therefore received a considerable amount of attention by the researchers. Flow, heat and mass transfer problems with chemical reaction are of great importance in many processes.

The effect of chemical reaction depends upon whether the reaction is homogeneous or heterogeneous. In well-mixed systems, the reaction is heterogeneous if it takes place in the interface and homogeneous if it takes place in solution. In most cases the reaction rates depends upon the concentration of species. A reaction is said to be of first order if the rate of reaction is directly proportional to the species concentration.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic chemical reactions. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers.

The composition of binary mixture is described by the concentration C, defined as the ratio of mass of rarer and lighter component to the total mass of the mixture in a given volume. The concentration C_1 of heavier and abundant component is given by C_1 = 1-C. A binary mixture subject to the temperature gradient can generate thermal diffusion i.e the temperature gradients cause solute fluxes. This phenomenon is known as Soret effect. In a binary fluid mixture the diffusion of individual species takes place by three mechanisms namely ordinary diffusion, baro-diffusion and thermo-diffusion. The diffusion flux ***i*** of the rarer and lighter component of the binary fluid mixture is given by Landau and Lifshitz [1] as

$$\boldsymbol{i} = -\rho D\left[\nabla C + k_p \nabla p + k_T \nabla T\right] \tag{1}$$

where D is the molecular diffusivity, $k_p D$ is the baro-diffusion coefficient, $k_T D$ is the thermal diffusion coefficient.

Many earlier researchers such as Howarth [2], Nigam [3] and Banks [4] studied the flow of an incompressible viscous fluid due to the rotation of a solid sphere. Singh [5], Siekann [6] and Banks [7] solved the problem of heat transfer in the flow of an incompressible viscous fluid due to rotation of a uniformly heated sphere, Banks [8] continued the study of the problem of laminar flow due to a rotating sphere numerically by using finite difference method, Kalita [9] studied this problem under the effect of a magnetic field and solved by Karman-Polhausen method, Sharma and Singh [10] discussed barodiffusion and thermal diffusion of a binary fluid mixture confined between two parallel discs in presence of a small axial magnetic field, Sharma and Singh [11] investigated separation of species of a binary fluid mixture confined between two concentric rotating circular cylinders in presence of a strong radial magnetic field, Sharma and Nath [12] studied numerically the effect of magnetic field on separation of binary mixture of viscous fluids by barodiffusion and thermal diffusion near a stagnation point, Sharma and Nath [13] continued the study of the effect of axial magnetic field on demixing of a binary fluid mixture due to the rotation of a heated sphere, Sharma and Konwar [14] discussed the effect of chemical reaction on mass distribution of a binary fluid mixture in unsteady MHD Couette flow, Sharma and Konwar [15] carried out the study on MHD flow, heat and mass transfer due to axially moving cylinder in presence of thermal diffusion, radiation and chemical reactions in a binary fluid mixture.

In the present paper we study the effects of heat generation and chemical reaction on heat and mass transfer of thermally and electrically conducting incompressible viscous binary fluid mixture due to a uniformly rotating heated sphere about a diameter in presence of magnetic field, baro-diffusion, thermal diffusion and viscous dissipation by considering the fluid mixture at infinity is at rest. The solutions of the boundary layer equations have been obtained by expanding the functions in ascending powers of $sin\theta$ and then solved numerically by using bvp4c.

FORMULATION OF THE PRBLEM

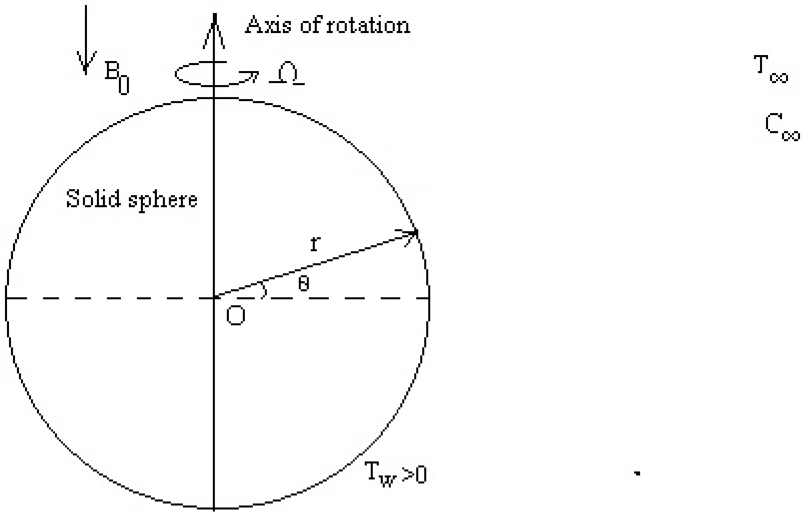

Fig.1 Physical model and co-ordinate system

Consider a steady, laminar and three-dimensional flow of an electrically conducting incompressible viscous binary fluid mixture due to a heated rotating sphere of radius $r = a$ about one of its diameters with a uniform angular velocity Ω. It is assumed that the temperature of the fluid mixture near the surface of the sphere T_w is higher than T_∞, the temperature of the fluid mixture at a large distance from the surface of the sphere. Let *u*, *v*, *w* are the velocity components in the increasing directions of the spherical polar co-ordinate (*r*, *θ*, *φ*) with *r* measured radially outward from the centre of the sphere, the angle *θ* is measured at the centre of the sphere from the axis of rotation and *φ* is the azimuthal angle. A magnetic field of strength B_0 is applied in a direction parallel to the axis of rotation. Since the motion is axi-symmetric, all the physical quantities are independent of *φ*.

Under the boundary layer approximations, the equations for conservation of mass, momentum, energy and species of the binary fluid mixture can be written as follows:

$$\frac{\partial u}{\partial r}+\frac{1}{a}\frac{\partial v}{\partial \theta}+\frac{v}{a}cot\,\theta=0 \qquad (2)$$

$$-\frac{1}{a}\left(v^2+w^2\right)=-\frac{1}{\rho}\frac{\partial p}{\partial r}-\frac{\sigma B_0{}^2}{\rho}v\,sin\,\theta\,cos\,\theta \qquad (3)$$

$$u\frac{\partial v}{\partial r}+\frac{v}{a}\frac{\partial v}{\partial \theta}-\frac{w^2}{a}cot\,\theta=\nu\frac{\partial^2 v}{\partial r^2}-\frac{\sigma B_0{}^2}{\rho}v\,cos^2\,\theta \qquad (4)$$

$$u\frac{\partial w}{\partial r}+\frac{v}{a}\frac{\partial w}{\partial \theta}+\frac{vw}{a}cot\,\theta=\nu\frac{\partial^2 w}{\partial r^2}-\frac{\sigma B_0{}^2}{\rho}w \tag{5}$$

$$\rho C_p\left(u\frac{\partial T}{\partial r}+\frac{v}{a}\frac{\partial T}{\partial \theta}\right)=k\frac{\partial^2 T}{\partial r^2}+\mu\left[\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{\partial w}{\partial r}\right)^2\right]+Q\left(T-T_\infty\right)+$$
$$\frac{\sigma B_0{}^2}{\rho}\left[v^2 cos^2\theta+w^2\right] \tag{6}$$

$$\left(u\frac{\partial C}{\partial r}+\frac{v}{a}\frac{\partial C}{\partial \theta}\right)=D\left[\frac{\partial^2 C}{\partial r^2}+\frac{\partial}{\partial r}\left(k_p\frac{\partial p}{\partial r}\right)+\frac{\partial}{\partial r}\left(k_T\frac{\partial T}{\partial r}\right)\right]-k_c\left(C-C_\infty\right) \tag{7}$$

with the boundary conditions

$$\left.\begin{aligned}&u=0,v=0,w=a\,\Omega\,sin\,\theta,T=T_W\,,\\&\rho Cu-\rho D\left(\frac{\partial C}{\partial r}+k_p\frac{\partial p}{\partial r}+k_T\frac{\partial T}{\partial r}\right)+\beta\sigma wB_0\,sin\,\theta=0\end{aligned}\right\}\text{ at } r=a \tag{8}$$

and

$$v\to 0,w\to 0,T\to T_\infty,C\to C_\infty \quad \text{at}$$

$$r\to\infty \tag{9}$$

where $k_p = AC$, $k_T = S_T C$, S_T is the Soret coefficient, A is a constant, σ is the electrical conductivity, β is the electrical characteristic constant of the conducting medium, ρ is the density, ν is the kinematic viscosity, k is the coefficient of thermal conductivity, k_c is the dimensional chemical reaction parameter, Q is the dimensional heat generation parameter, μ is the viscosity, C_p is the specific heat at constant pressure, T is the temperature, p is the pressure of the binary fluid mixture, C is the concentration of the rarer and lighter component at any point and C_∞ is the fixed value of the concentration of the rarer and lighter component of the binary fluid mixture in its undisturbed state at infinity respectively.

METHOD OF SOLUTION

We shall adopt the power series expansions for u, v, w, T, p and C as follows:

$$u=-\sqrt{\nu\,\Omega}\left[2F_1+\left(4F_3-3F_1\right)sin^2\theta+\left(6F_5-5F_3\right)sin^4\theta+...\right] \tag{10}$$

$$v=\frac{a\Omega}{2}sin\,2\theta\left[F_1'+F_3'\,sin^2\theta+F_5'\,sin^4\theta+...\right] \tag{11}$$

$$w = a\Omega \sin\theta \left[G_1 + G_3 \sin^2\theta + G_5 \sin^4\theta + ... \right] \tag{12}$$

$$p = \rho a \Omega^2 \sqrt{\frac{\nu}{\Omega}} \left[p_0 + p_1 \sin^2\theta + p_3 \sin^4\theta + ... \right] \tag{13}$$

$$T = T_\infty + \left(T_w - T_\infty \right) \left[T_1 + T_3 \sin^2\theta + T_5 \sin^4\theta + ... \right] \tag{14}$$

$$C = C_\infty \left[\psi_0 + \psi_1 \sin^2\theta + \psi_3 \sin^4\theta + ... \right] \tag{15}$$

where F_i, G_i, p_i, T_i and ψ_i (i = 0, 1, 2, 3…) are functions of the independent non dimensional variable η given by the expression

$$\eta = \sqrt{\frac{\Omega}{\nu}} \left(r - a \right) \tag{16}$$

With the help of the above expansions the equation of continuity Eq. (2) is identically satisfied. Substituting the expansions Eqs. (10-13) into Eqs. (3-5) and equating similar terms on both sides of the equations the following non linear coupled ordinary differential equations are obtained.

$$p_0' = 0 \tag{17}$$

$$p_1' = \left(F_1' \right)^2 + \left(G_1 \right)^2 - M F_1' \tag{18}$$

$$F_1''' = \left(F_1' \right)^2 - 2 F_1 F_1'' - \left(G_1 \right)^2 + M F_1' \tag{19}$$

$$F_3''' = 3F_1F_1'' - 2F_1F_3'' - 4F_3F_1'' + 4F_3'F_1' - 2\left(F_1' \right)^2 - 2G_1G_3 + M \left\{ F_3' - F_1' \right\} \tag{20}$$

$$G_1'' = 2F_1'G_1 - 2F_1G_1' + MG_1 \tag{21}$$

$$G_3'' = 3F_1G_1' - 4F_3G_1' - 2F_1G_3' + 2F_3'G_1 + 4F_1'G_3 - 2F_1'G_1 + MG_3 \tag{22}$$

We shall use the expression $\bar{T} = \dfrac{T - T_\infty}{T_w - T_\infty}$ in the Eq. (6) to make it non-dimensional and we obtain as follows:

$$-u\frac{\partial \bar{T}}{\partial \eta}+v\frac{\partial \bar{T}}{\partial \theta}=\frac{1}{Pr}\frac{\partial^2 \bar{T}}{\partial \eta^2}+Ec\left\{\left(\frac{\partial v}{\partial \eta}\right)^2+\left(\frac{\partial w}{\partial \eta}\right)^2\right\}+\delta \bar{T}+MEc\left(v^2 cos^2\theta+w^2\right) \tag{23}$$

Suppressing bars we get

$$-u\frac{\partial T}{\partial \eta}+v\frac{\partial T}{\partial \theta}=\frac{1}{Pr}\frac{\partial^2 T}{\partial \eta^2}+Ec\left\{\left(\frac{\partial v}{\partial \eta}\right)^2+\left(\frac{\partial w}{\partial \eta}\right)^2\right\}+\delta T+MEc\left(v^2 cos^2\theta+w^2\right) \tag{24}$$

Now applying the expansion Eq.(14) into the Eq.(24) and equating similar terms on both sides we get

$$T_1''=-Pr\left(2F_1T_1'+\delta T_1\right) \tag{25}$$

$$T_3''=Pr\Big[2F_1'T_3+3F_1T_1'-2F_1T_3'-4F_3T_1'-Ec\left\{\left(F_1''\right)^2+\left(G_1'\right)^2\right\}-\delta T_3-$$
$$MEc\left\{\left(F_1'\right)^2+\left(G_1\right)^2\right\}\Big] \tag{26}$$

The following expressions Eqs.(27-28) are obtained by using the expansion Eq.(15) into the equation (7) and equating similar terms on both sides we get

$$\psi_0''=\gamma Sc\left(\psi_0-1\right)-2ScF_1\psi_0'-Bd\left(\psi_0 p_0''+\psi_0' p_0'\right)-td\left(\psi_0 T_1''+\psi_0' T_1'\right) \tag{27}$$

$$\psi_1''=\gamma Sc\psi_1-Sc\left(2F_1\psi_1'+4F_3\psi_0'-3F_1\psi_0'-2F_1'\psi_1\right)-$$
$$Bd\left(\psi_0 p_1''+\psi_1 p_0''+\psi_1' p_0'+\psi_0' p_1'\right)-td\left(\psi_0 T_3''+\psi_1 T_1''+\psi_1' T_1'+\psi_0' T_3'\right) \tag{28}$$

New boundary conditions are

$$\left.\begin{array}{l} F_1=F_3=0, F_1'=F_3'=0, G_1=1, G_3=1, T_1=1, T_3=0 \\ \psi_0'+td\psi_0 T_1'=0, \\ \psi_1'+Bd\left[\psi_0\left\{\left(F_1'\right)^2+\left(G_1\right)^2\right\}-M\psi_0 F_1'\right]+ \\ td\left(\psi_0 T_3'+\psi_1 T_1'\right)-LG_1=0 \end{array}\right\} \text{ when } \eta=0 \tag{29}$$

and

$$F_1 = F_3 = 0, G_1 = 0, G_3 = 0, T_1 = 0, T_3 = 0, \psi_0 = 1, \psi_1 = 0 \text{ when } \eta \to \infty \tag{30}$$

where $M = \dfrac{\sigma B_0^{\,2}}{\rho\Omega}$, $Pr = \dfrac{\mu C_p}{k}$, $\delta = \dfrac{Q}{\rho C_p \Omega}$, $Ec = \dfrac{a^2\Omega^2}{C_p\left(T_W - T_\infty\right)}$,

$$Bd = \rho A a \Omega^2 \sqrt{\frac{\gamma}{\Omega}}, td = S_T\left(T_w - T_\infty\right), Sc = \frac{\nu}{D}, L = \frac{a\Omega\beta\sigma B_0}{C_\infty \rho D}\sqrt{\frac{\nu}{\Omega}}$$

and $\gamma = \dfrac{k_c}{\Omega}$

are magnetic parameter, Prandtl number, heat generation parameter, Eckert number, baro-diffusion parameter, thermal diffusion parameter, Schmidt number, electrical conductivity parameter and chemical reaction parameter respectively. Here prime denotes differentiation with respect to η.

Since the solutions of non-linear coupled ordinary differential Eqs.(17-22) and Eqs.(25-28) under the boundary conditions Eqs. (29-30) cannot be obtained in closed form therefore we have solved these equations numerically with Matlab's boundary value problem solver bvp4c.

From the process of numerical computation the local skin friction, the local Nusselt number and the local Sherwood number, which are respectively proportional to $F_1''(0), F_3''(0)$, $G_1'(0), G_3'(0), T_1'(0), T_3'(0), \psi_0'(0), \psi_1'(0)$ for various values of magnetic parameter *M*, heat generation parameter δ and chemical reaction parameter γ are presented in tabular form.

RESULTS AND DISCUSSIONS

Numerical calculations have been carried out at $\theta = 15^o$ for various values of the chemical reaction parameter γ and heat generation parameter δ.

Figs.2 and 3 depict the temperature and concentration profiles for various values of the heat generation parameter δ by taking δ = [0, 0.1, 0.2]; *M* =1; *Sc* = 4; *Pr* = 1.1; *E c*= 0.01; *Bd* = 0.001; *td* = 0.001; γ = 0.5; *L* = 4.

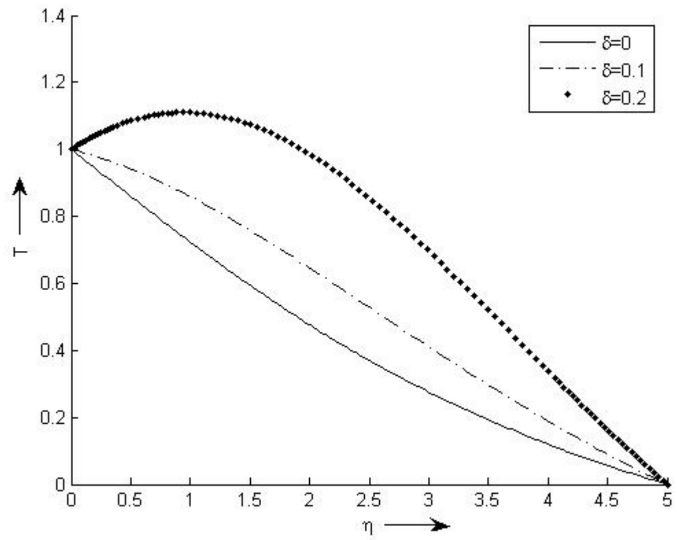

Fig2: Temperature profiles for different values of δ against η

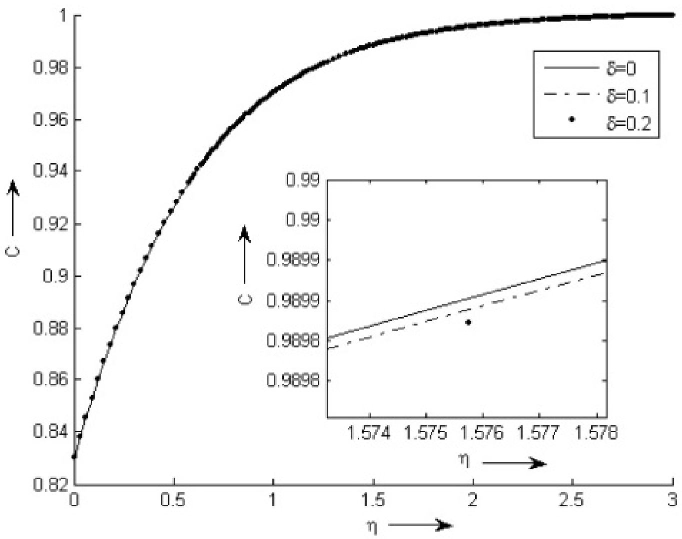

Fig3: Concentration profiles for different values of δ against η

It has been noticed from the Fig. 2 that the temperature of the binary fluid mixture is more near the surface of the sphere and less towards the end of the boundary layer. The effect of the increase in the values of the heat generation parameter δ increases the temperature of the binary fluid mixture at any point in the boundary layer. This increase in the temperature has the tendency to increase the thermal buoyancy force. It is also observed that the temperature profile overshoots in the immediate surrounding area of the surface of the sphere for higher values of heat generation parameter δ.

From the Fig.3 it is clear that the concentration of the rarer and lighter component of the binary fluid mixture in the immediate vicinity of the surface of the sphere is less than the concentration away from the surface of the sphere. Moreover, as the values of the heat generation parameter δ increases the concentration of the rarer and lighter component of the binary fluid mixture decreases. This change is found to be negligible.

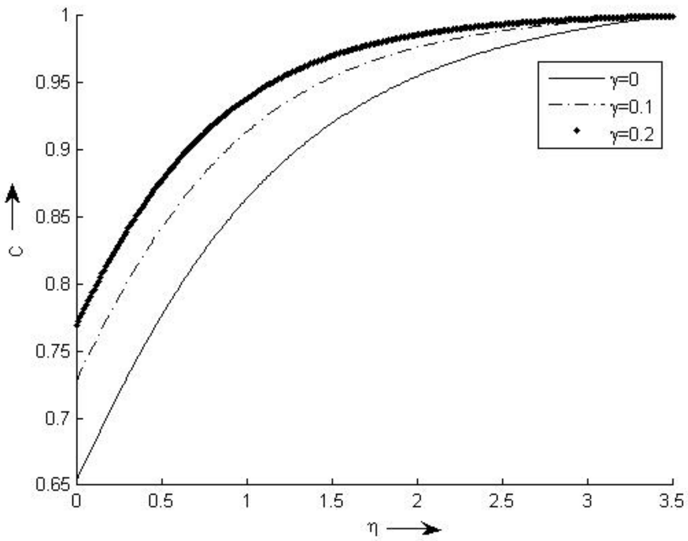

Fig4: Concentration profiles for different values of γ against η

Fig.4 represents the concentration profiles for different values of γ against η by taking γ = [0, 0.1, 0.2]; M = 1; Sc = 4; Pr = 1.1; Ec = 0.01; Bd = 0.001; td = 0.001; δ = 0.4; L = 4. Fig.4 depicts that the concentration of the rarer and lighter component of the binary fluid mixture in the immediate vicinity of the surface of the sphere is less than the concentration away from the surface of the sphere. Also it is clearly seen that with the increase in the values of the chemical reaction parameter γ increases the concentration of the rarer and lighter component of the binary fluid mixture at any point in the boundary layer. Variation of the chemical reaction parameter γ does not show any impact on the temperature of the binary fluid mixture, so pictorial representation is ignored.

Table 1

	Pr=1.1; *Sc*=4; *L*=4; *Ec*=0.01; *Bd*=0.001; *td*=0.001; δ=0.4; γ =0.5;			*Pr*=1.1; *Sc*=4; *L*=4; *Ec*=0.01; *Bd*=0.001; *td*=0.001; *M*=1; γ =0.5;			*Pr*=1.1; *Sc*=4; *L*=4; *Ec*=0.01; *Bd*=0.001; *td*=0.001; *M*=1; δ =0.4;			*Pr*=1.1; *Sc*=4; *L*=4; *Bd*=0.001; *td*=0.001; *M*=1; δ =0.4; γ =0.5;		
	M			δ			γ			*Ec*		
	0.1	**0.4**	**0.8**	**0.03**	**0.05**	**0.07**	**0.3**	**0.6**	**1**	**0.1**	**0.5**	**0.9**
F_1''	0.4744	0.4029	0.3343	0.3090	0.3090	0.3090	0.3090	0.3090	0.3090	0.3090	0.3090	0.3090
F_3''	0.1038	0.0828	0.0641	0.0581	0.0581	0.0581	0.0581	0.0581	0.0581	0.0581	0.0581	0.0581
G_1'	-0.6613	-0.8030	-0.9841	-1.0691	-1.0691	-1.0691	-1.0691	-1.0691	-1.0691	-1.0691	-1.0691	-1.0691
G_3'	0.1257	0.0862	0.0540	0.0432	0.0432	0.0432	0.0432	0.0432	0.0432	0.0432	0.0432	0.0432
T_1'	0.0468	0.1045	0.1602	-0.1600	-0.0964	-0.0148	0.8223	0.8223	0.8223	2.5178	2.5178	2.5178
T_3'	0.1234	0.1023	0.0776	0.0475	0.0490	0.0507	0.0986	0.0986	0.0986	0.1991	0.6846	1.1701
ψ_0'	-0.1745	-0.2792	-0.3842	0.1599	0.0964	0.0148	-0.8222	-0.8223	-0.8224	-0.2230	-0.2230	-0.2230
ψ_1'	3.9993	3.9995	3.9999	3.9985	3.9987	3.9989	4.0014	4.0008	4.0005	3.9994	3.9990	3.9985

CONCLUSIONS

From the above discussions we can conclude that

1. The effect of increase in the values of the heat generation parameter δ increases the temperature and decreases the concentration of the rarer and lighter component of the binary fluid mixture at any point in the boundary layer.
2. The effect of increase in the values of the chemical reaction parameter γ increases the concentration of the rarer and lighter component of the binary fluid mixture at any point in the boundary layer.

REFERENCES

[1] L.D. Landau, E.M. Lifshitz, 1959, “Fluid Mechanics”, 2nd edn. London. Pergamen Press.

[2] L.Howarth, 1951. “The boundary layer in three dimensional flow Part I”, Phil. Mag. Ser. 7, 42, 239.

[3] S.D.Nigam, 1954. “Note in the boundary layer on a rotating sphere”, ZAMP, 5, 151-155.

[4] W.H.H. Banks, 1965, “The boundary layer on a rotating sphere”, Quart. Jour. Mech. Appl. Maths, 18(4), 443-454.

[5] S.N.Singh, 1960, “Heat transfer by laminar flow from a rotating sphere”, Appl. Sci. Res., A9, 197-205.

[6] I. Siekann, 1962, “The calculation of the thermal boundary layer on a rotating sphere”, ZAMP, 13, 468-482.

[7] W.H.H. Banks, 1965, "The thermal laminar boundary layer on a rotating sphere", ZAMP, 16 (6), 780-788.
[8] W.H.H. Banks, 1976, "The laminar boundary layer on a rotating sphere", Acta Mechanica, 24 (3-4), 273-287.
[9] K.C. Kalita, 1982, "Dibrugarh University Ph.D. Thesis".
[10] B.R.Sharma and R.N. Singh, 2008, "Barodiffusion and thermal diffusion of a binary fluid mixture confined between two parallel discs in presence of a small axial magnetic field". Latin American Applied Research, 38, 313-320.
[11] B.R.Sharma and R.N. Singh, 2010, "Separation of species of a binary fluid mixture confined between two concentric rotating circular cylinders in presence of a strong radial magnetic field", Heat Mass Transfer, 46, 769-777.
[12] B.R. Sharma and Kabita Nath, 2012, "The effect of magnetic field on separation of binary mixture of viscous fluids by barodiffusion and thermal diffusion near a stagnation point-a numerical study", Int. Jour. Mathematical Archive, 3(3), 1118-1124.
[13] B.R.Sharma and Nath, Kabita, 2012, "Effect of axial magnetic field on demixing of a binary fluid mixture due to the rotation of a heated sphere" Int. Journal of Computer Applications, 53(10), 47-52.
[14] B.R.Sharma and Hemanta Konwar, 2014, "Effect of chemical reaction on mass distribution of a binary fluid mixture in unsteady MHD Couette flow", IJIRSET, 3(8), 15545-15552.
[15] B.R.Sharma and Hemanta Konwar, 2015, "MHD flow, heat and mass transfer due to axially moving cylinder in presence of thermal diffusion, radiation and chemical reactions in a binary fluid mixture", IJCA, 110 (15), 52-59.