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Effect Of Uniform Magnetic Field On The Motion Of Porous Sphere In Spherical Container

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Abstract:

This paper concerns the motion of porous sphere in a spherical container in presence of uniform magnetic field. For inside the porous sphere Brinkman's equation and outside the porous sphere Stokes equation with their stream function formulation is used. The force experienced by porous sphere and wall correction factor is evaluated. Explicit expressions of stream functions in presence of uniform magnetic field are obtained for both the inside and outside flow fields. The variation of drag coefficient and wall correction factor with respect to different parameter is presented graphically.

Keywords: Drag force, Magnetic field, Porous medium, Modified Bessel function, Permeability

Introduction:

Many process and phenomena in Science and Engineering and Technology involve the motion of flow fluids past and through porous media. For example in membrane filtration, the product flow and membrane life span are directly affected by hydrodynamic properties of the porous medium such as permeability. Therefore study of flow fluids past and through porous media is very important for real time applications. The motion of particles in a container is important because it gives information on wall effects. Many researchers have considered the motion of solid or fluid sphere in a spherical container. Cunnigham [1] considered the motion of solid spherical particle in a spherical container. Ramkisson [2]considered the Reiner-Revlin fluid sphere in a spherical container. Recently Srinivascharya [3] solved the problem of motion of porous sphere in a spherical container. Hydrodynamic permeability of membrane composed of porous spherical particles in the presence of uniform magnetic field studied by Srivastava et. al [4]. In this paper he studied the hydrodynamic permeability using cell model technique.

The Present paper is the extension of the work done by Srinivascharya. In this paper we introduced uniform magnetic field in the motion of porous sphere in spherical container. The force experienced by porous sphere and wall correction factor is evaluated. Explicit expressions of stream functions in presence of uniform magnetic field are obtained for both the inside and outside flow fields. The variation of drag coefficient and wall correction factor with respect to different parameter is presented graphically. The previous result obtained by researchers have verified graphically.

Formulation of the problem:

In this mathematical model we consider a porous spherical particle of radius a passing the center of a spherical vessel of radius b containing an incompressible Newtonian viscous fluid. The porous medium is assumed to be homogeneous and isotropic. Further let porous spherical particle is stationary and steady axisymmetric Stokes flow of an electrically conducting viscous incompressible fluid in presence of uniform magnetic field in transverse direction has been established around and through it with an uniform velocity U with positive Z-axis.

The equations of motion for the region outside the porous sphere $(a \leq r \leq b)$ is governed by Stokes equation

$$\mu_h^2 \sigma^{(2)} \left(v^{(2)} \times H \right) \times H + \mu^{(2)} \nabla^2 v^{(2)} = \nabla p^{(2)} \qquad (1)$$

$$\nabla . v^{(2)} = 0 \qquad (2)$$

Here the magnetic Reynolds number is assumed to be very small and there is no external electric field so that induced current is very small and hence, it can be neglected.

For the region inside the porous sphere $(r \leq a)$ we assumed that flow is governed by the Brinkman equation with continuity condition

$$-\frac{\mu^{(1)}}{k} v^{(1)} + \mu_h^2 \sigma^{(1)} \left(v^{(1)} \times H \right) \times H + \mu^{(1)} \nabla^2 v^{(1)} = \nabla p^{(1)} \qquad (3)$$

$$\nabla . v^{(1)} = 0 \qquad (4)$$

Where $v^{(1)}, v^{(2)}$ and $p^{(1)}, p^{(2)}$ are velocity and pressure in the inside and outside of porous region. $\mu^{(1)}, \mu^{(2)}$ and $\sigma^{(1)}, \sigma^{(2)}$ are viscosities and electrical conductivity of fluid for inside and outside of porous region. μ_h is the magnetic permeability, which is supposed to be same for clear fluid and porous medium, and H is transverse magnetic field vector to the fluid velocity.

For simplicity here we assumed that. $\mu^{(1)} = \mu^{(2)} = \mu$ and $\sigma^{(1)} = \sigma^{(2)} = \sigma$

Therefore governing equation becomes as

$$\mu_h^2 \sigma \left(v^{(2)} \times H \right) \times H + \mu \nabla^2 v^{(2)} = \nabla p^{(2)} \qquad (5)$$

$$-\frac{\mu}{k} v^{(1)} + \mu_h^2 \sigma \left(v^{(1)} \times H \right) \times H + \mu \nabla^2 v^{(1)} = \nabla p^{(1)} \qquad (6)$$

Solution of the problem:

Introducing the Stream function $\Psi^{(i)}(r, \theta)$; $i = 1, 2$ for both the regions through

$v_r^{(i)} = -\frac{1}{r^2 \sin\theta}\frac{\partial\Psi^{(i)}}{\partial\theta}, v_\theta^{(i)} = \frac{1}{r\sin\theta}\frac{\partial\Psi^{(i)}}{\partial r}; i = 1, 2$ (7)

In equation() and() and eliminating pressure from the resulting equations we get following dimensionless equation for $\Psi^{(i)}(r,\theta); i = 1, 2$

$E^4\Psi^{(1)} - m^2E^2\Psi^{(1)}=0 ; 1 \leq r \leq \frac{1}{\eta}$ (8)

& $E^4\Psi^{(2)} - s^2E^2\Psi^{(2)}=0; 0 \leq r \leq 1$ (9)

Where $E^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}$

$M^2 = \frac{\mu_h^2\sigma\mu_0^2a^2}{\mu}$; $S^2 = M^2 + \frac{1}{k}$ is porosity parameter and k is dimensionless specific permeability of porous sphere.

The boundary conditions are:

$v_r^{(1)} = v_r^{(2)}$ on $r = a$

$v_\theta^{(1)} = v_\theta^{(2)}$ on $r = a$

$T_{r\theta}^{(1)} = T_{r\theta}^{(2)}$ on $r = a$

$p^{(1)} = p^{(2)}$ on $r = a$

On the outer sphere the condition of impenetrability leads to

$v_r^{(1)} = U\cos\theta, v_\theta^{(1)} = -U\sin\theta$ on $r = b$

The boundary conditions in terms of stream functions are

$\Psi^{(1)}(r,\theta) = \frac{1}{2}r^2Sin^2\theta, \Psi_r^{(1)} = rSin^2\theta \; on \; r = \frac{1}{\eta}$ (10)

And

$\Psi^{(1)}(r,\theta) = \Psi^{(2)}(r,\theta), \Psi_r^{(1)}(r,\theta) = \Psi_r^{(2)}(r,\theta)$ (11)

$\Psi_{rr}^{(1)}(r,\theta) = \Psi_{rr}^{(2)}(r,\theta), p^{(1)}(r,\theta) = p^{(2)}(r,\theta) \; on \; r = 1$ (12)

Where $\eta = \frac{a}{b}$

The solution of equation (8) and (9) which are non singular everywhere in the flow region, are

$\Psi^{(1)}(r,\zeta) = \left[\frac{A_1}{r} + B_1r^2 + C_1\sqrt{r}I_{-\frac{3}{2}}(Mr) + D_1\sqrt{r}I_{\frac{3}{2}}(Mr)\right]G_2(\zeta)$ (13)

And

$\Psi^{(2)}(r,\zeta) = \left[B_2r^2 + D_2\sqrt{r}I_{\frac{3}{2}}(Sr)\right]G_2(\zeta)$ (14)

Where $I_{\pm v}(Mr) \; and \; I_{\pm v}(Sr)$ both are the modified Bessel function of the first kind and of non integer index $v = \{m(m-1) + \frac{1}{4}\}^{1/2}$; $G_n(\zeta)$ is the Gegenbauer function of the first kind of order n and degree -1/2.

Using boundary condition given from equations (10)-(12) in equations (13) and (14), we get all arbitrary constant appearing in stream function.

Drag on the body and wall effects:

The drag experienced by the inner spherical particle is given by

$$F = \mu\pi ua\int_0^\pi \varpi^3\frac{\partial}{\partial r}\left(\frac{E^2\Psi}{\varpi^2}\right)rd\theta$$

Where $\bar{\omega} = r sin\theta$.on putting the value of $E^2\Psi$ and after integration we get the value of drag force as

$$F = F_o\,[(M(-3D_1M + (-2 + M^2)C_1)\cosh M + (-2C_1(-4 + M^2) + M(3 + M^2)D_1)\sinh M]$$

where $F_o = -\frac{2}{3}\pi\mu aUM$

Also, the drag coefficient is defined as

$$C_D = \frac{F}{1/2\rho\pi\mu a^2} = G_o\,[(M(-3D_1M + (-2 + M^2)C_1)\cosh M + (-2C_1(-4 + M^2) + M(3 + M^2)D_1)\sinh M]$$

where $G_o = -\frac{4UM}{3a\rho}$.

Numerical Results with Explanation:

We provide the numerical estimation for drag force versus porosity parameter (s) with and without magnetic field. In Fig.1 we compare the effect due to the inclusion of magnetic field which explains the dotted line without magnetic field and bold line with magnetic field. In the presence of magnetic field it is clearly visible that drag force increases. In Figure 2 we take a constant magnetic field and compared the drag force for four different porous parameters. Here we see that increasing inner porous particle size keep reduces the drag force along porous parameter. Next we move to analyze the effect of drag coefficient with increasing magnetic field in Figure 3. It is evident that the increasing magnetic field decreases the drag coefficient.

Later we checked the result for wall factor dependence on inner porous particle size. In Figure 4. We vary the porosity parameter from 0.7 to 1.8 to see the effect on wall factor. It shows the increasing feature of wall factor. It is worth noticing that here we keep the magnetic field high. A novel feature turns out as we change the magnetic field to some low critical value (Figure 5), the wall factor depicts well like structure. This feature has not been studied earlier and we hope to analyze this too much detail in future.

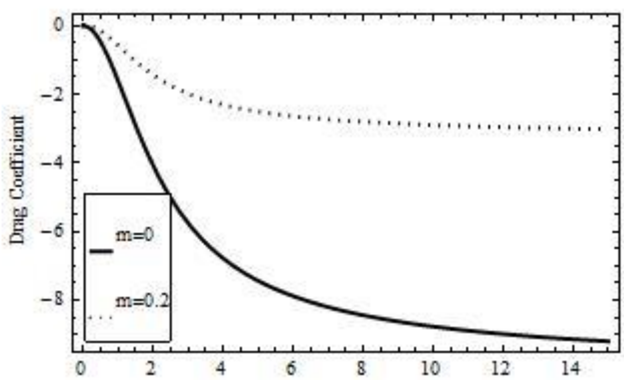

Figure 1. Drag Coefficient verses S with and without magnetic field

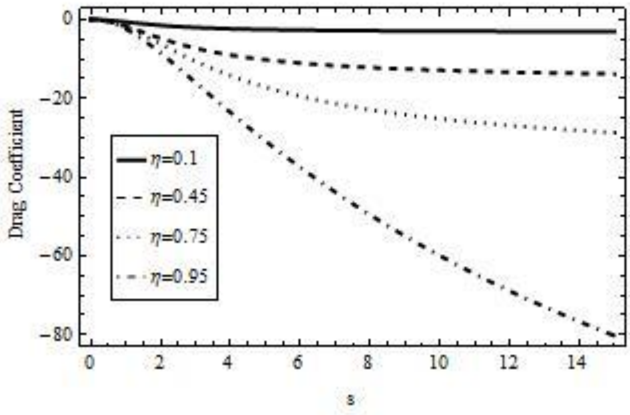

Figure2: Drag Coefficient verses S with four different porosity

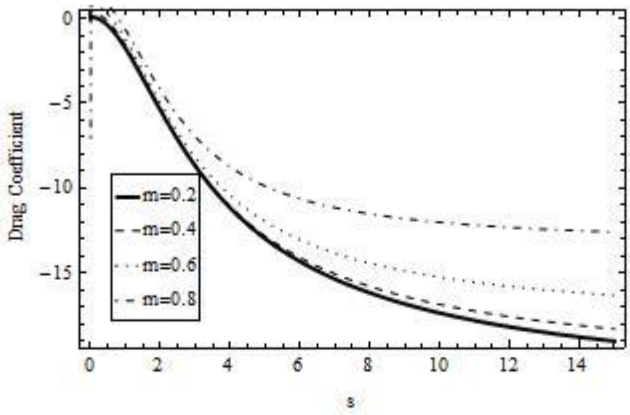

Figure 3: Drag Coefficient verses S with four different magnetic field

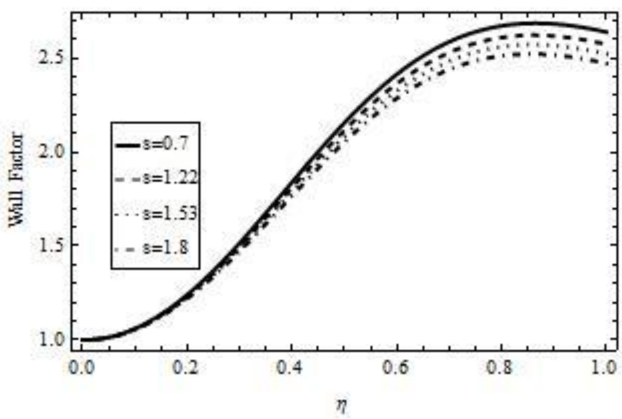

Figure 4: Wall factor verses η for four different porosity

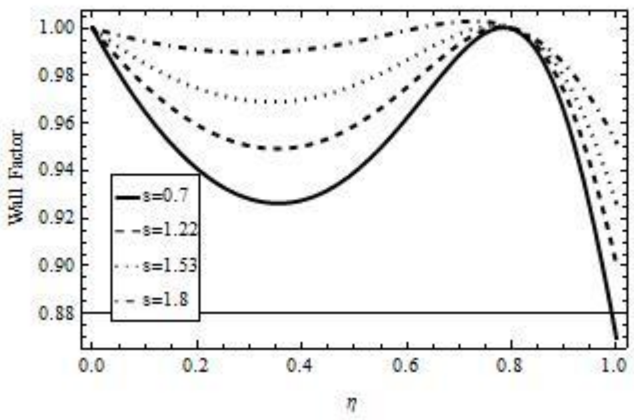

Figure 5: Wall factor verses η for four different s

Conclusion:

In this paper we derive the expression of stream function in presence of uniform magnetic field for the reign inside the porous sphere and outside the porous sphere. Also we find the drag coefficient and wall correction factor for the porous sphere and result has been presented graphically which agree with some of the previous result of Srinivascharya.

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