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Slip effect on Casson flow of blood

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Abstract:

A mathematical model is presented here to study the slip effect on Casson flow of blood. Here I have considered one dimensional steady flow of blood through an axially symmetric but radially non-symmetric stenosed artery, where blood behaves like a Casson fluid. The variations of axial velocity, plug velocity, flow rate, wall shear stress and pressure gradient have been incorporated. The results are shown graphically and discussed.

Key words: Casson fluid, slip velocity, stenosis, flow rate, wall shear stress.

Introduction:

Stenosis is formed by the accumulation of fats/cholesterol on the arterial wall and proliferation of the connective tissue. It is already known that due to the presence of stenosis in the lumen of the artery, the nature of blood flow changes from its usual state to a distributed flow condition. Many Cardiovascular diseases such as blood pressure, atherosclerosis, heart attack and brain haemorrhage are influenced by the presence of the arterial stenosis.Stenosis is one of the most wide-spread arterial disease. Effect of stenosis on cardiovascular system has been determined by studying the flow characteristics of blood in stenotic region of artery.

In view of this some mathematicians have presented various mathematical models to study the blood flow characteristics through stenosed artery (Young[1], Lee and Fung[2], Shukla et. al [3], Chaturani and Samy [4] and Radhakrishnamacharya et.al [5], Verma and Parihar[6], Biswas and Chakraborty [7]) by considering the blood as a Newtonian fluid. But since it has been observed that whole blood being permanently a suspension of erythrocytes in plasma, Majhi and Nair [8] suggested that under certain conditions blood behaves like a non-Newtonian fluid. In view of this many researchers study the power law fluid model of blood by giving reason that under certain conditions blood behaves like a power law fluid.(Sanjeev Kumar et, al [9], Singh and Shah [10]).

Some authors have analysed mathematical models (Maruthiprasad and Radhakrishn-amacharya [11], Maruthiprasad et.al [12], Siddique et.al [13]) by considering blood as a Herschel-Bulkley type non-Newtonian fluid. Biswaset. al [14] have studied two layered pulsatile flow of blood through an arterial tube by considering the core layer as Bingham plastic type fluid and the peripheral layer as Newtonian fluid.Sanyal and Maiti [15] have investigated two layered mathematical model by taking both the layers as Herschel- Bulkley type non-Newtonian fluid. Many authors have used the Casson fluid model for mathematical modelling of blood flow (Blair [16], Charm and Kurland [17], Aroesty and Gross [18]). Chaturani and Samy [19] analyzed the pulsatile flow of Casson fluid through stenosed artery using perturbation method. A mathematical model model of blood flow through an irregular arterial mild stenosis is developed by Jain et.al [20] and they have observed that if the viscosity of fluid increases the velocity of fluid decreases in presence of arterial stenosis. Biswas and Laskar [21] have presented a mathematical to study the steady flow of blood through a stenosed artery.

In the present analysis I propose to discuss the slip effect on Casson flow of blood through constricted artery.

Mathematical Formulation:

In this study, I have considered the steady flow of blood through an axially symmetric but radially non-symmetric stenosed artery.

The geometry of stenosis is taken as (Biswas [21])

$$\begin{aligned} R(z) &= R_0 - \frac{\delta}{2}\left[\, 1+ \cos\frac{2\pi}{L_0}(z-d-L_0)\right]; \quad d \le z \le d+L_0 \\ &= R_0, \quad \text{otherwise}, \end{aligned} \tag{1}$$

where R_0 , the radius of the tube ; R(z), the radius in the stenotic region; L_0 , the stenosis length, d indicates its location and δ be the maximum height of the stenosis , L be the length of the artery.

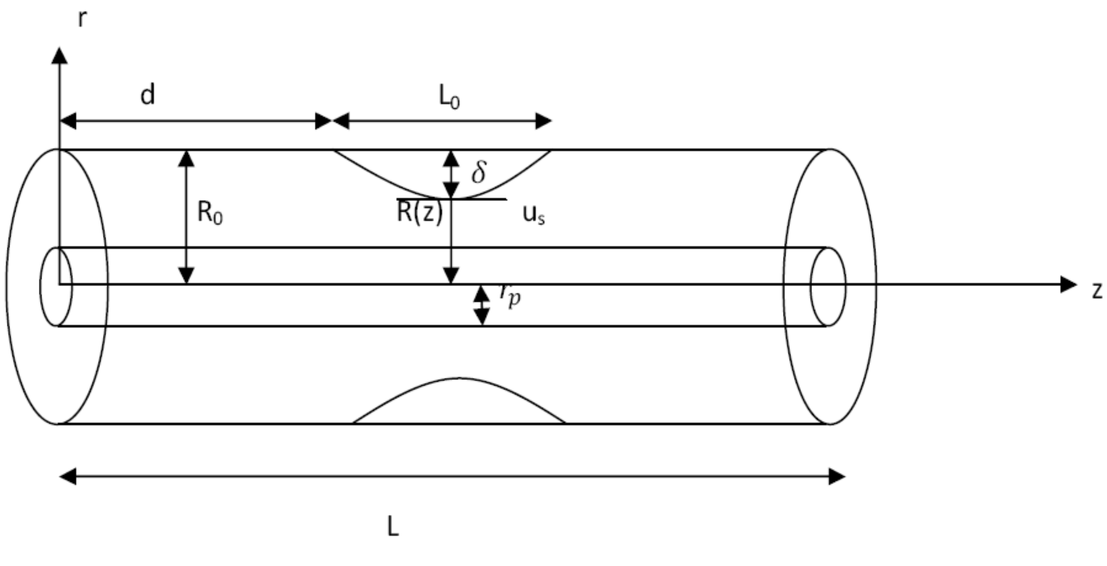

Fig 1: Geometry of Flow and Coordinate System

Problem and Solution:

The equation governing the motion is given by

$$0 = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) \tag{2}$$

$$0 = \frac{\partial p}{\partial r} \tag{3}$$

in which τ represents the shear stress of blood considered as Casson fluid and p, the pressure at any point.

The relationship between shear stress and shear rate is given by

$$-\frac{\partial u}{\partial r} = \frac{1}{k}(\sqrt{\tau} - \sqrt{\tau_c})^2 \text{ , } \tau \geq \tau_c \tag{4}$$

$$= 0, \qquad \tau < \tau_c \tag{5}$$

where u stands for the axial velocity of blood, τ_c is the yield stress and k is the coefficient of viscosity.

The boundary conditions are

$$u = u_s \text{ at } r = R(z) \tag{6}$$

$$\tau \text{ is finite at } r = 0, \tag{7}$$

whereu_s is the axial slip velocity.

Integrating (2) and using the boundary condition (7) we get

$$\tau = -\frac{r}{2}\frac{dp}{dz} \tag{8}$$

The skin-friction τ_R is given by

$$\tau_R = -\frac{R}{2}\frac{dp}{dz} \text{ , } \quad R = R(z) \tag{9}$$

From (8) and (9)

$$r_p = \frac{2\tau_c}{P} \text{ , } \tau_R = \frac{RP}{2}$$

$$\frac{r_p}{R} = \frac{\tau_c}{\tau_R} = \beta \text{, say} \tag{10}$$

Integrating equation (4) and using the boundary condition (7) the velocity function is given by

$$u = u_s + \frac{1}{k}\left[\frac{\tau_R}{2R}(R^2 - r^2) + \tau_c(R - r) - \frac{4}{3}\sqrt{\left(\frac{\tau_R\tau_c}{R}\right)}(R^{3/2} - r^{3/2})\right]$$

$$= u_s + \frac{\tau_{R.R}}{k}\left[\frac{1}{2}\left(1 - \frac{r^2}{R^2}\right) + \beta\left(1 - \frac{r}{R}\right) - \frac{4}{3}\sqrt{(\beta)}\left(1 - \frac{r^{3/2}}{R^{3/2}}\right)\right] \tag{11}$$

The expression for plug velocity is

$$u_p = u_s + \frac{\tau_{R.R}}{k}\left(\frac{1}{2} - \frac{4}{3}\sqrt{\beta} + \beta - \frac{1}{6}\beta^2\right) \tag{12}$$

The volumetric flow rate i.e, the flux is given by

$$Q = \int_0^R 2\,\pi r u\, dr$$

$$= \pi R^2 [u_s + \frac{\tau_{R.R}}{k}(\frac{1}{4} + \frac{\beta}{3} - \frac{4}{7}\sqrt{\beta})]$$

When $\frac{\tau_c}{\tau_R} \ll 1$, replacing $\frac{1}{3}$ by $\frac{16}{49}$ in the second term we get

$$Q = \pi R^2 [u_s + \frac{\tau_{R.R}}{k}(\frac{1}{2} - \frac{4}{7}\sqrt{\beta})^2]$$

From above we get

$$\frac{Q}{\pi R^2} - u_s = \frac{R}{k}(\frac{\sqrt{\tau_R}}{2} - \frac{4}{7}\sqrt{\tau_c})^2$$

Thus $\tau_R = \frac{16}{49}\tau_c + \frac{4k}{R}(\frac{Q}{\pi R^2} - u_s) + \frac{16}{7}\sqrt{\frac{\tau_{ck}}{R}}\sqrt{\frac{Q}{\pi R^2} - u_s}$

In the absence of stenosis the expression for wall shear stress becomes

$$\tau_N = \frac{16}{49}\tau_c + \frac{4k}{R_0}(\frac{Q}{\pi {R_0}^2} - u_s) + \frac{16}{7}\sqrt{\frac{\tau_{ck}}{R_0}}\sqrt{\frac{Q}{\pi {R_0}^2} - u_s}$$

which represents the skin friction for a normal arterial segment.
The non-dimensional expression for wall shear stress may be put as

$$\overline{\tau} = \frac{\tau_R}{\tau_N}$$

Numerical Computations:

To illustrate the flow characteristics the results are shown graphically with the help of MATLAB – 7.6. To attain the numerical results for axial velocity, plug velocity, flux and the wall shear stress, some parameters have been taken constant with the values

L_0= 1, d = 0.5, k = 2, $\tau_c = 0.1$, $R_0 = 1.5$, u_s = 0.05

Figures 2,3 give the variation of axial velocity for different values of τ_c and u_s with the variation of r. It is observed that axial velocity u decreases with the increase of r but the reverse effect occurs when τ_c and u_s increases.
Figures 4,5 show the variation of plug velocity u_p for different values of τ_c and u_s with the variation of z. It is found that u_p decreases up to the zero values of z and then increases. $u_p$decreases with the increase of τ_c but increases with the increase ofu_s.
Figures 6,7 illustrate the behaviour of flux Q for different values of τ_c and u_s with the variations of z. It is observed that Q increases up to the zero values of z and then decreases. Q increases with both the increase of $\tau_c$and u_s .
Figures 8,9 depict the variation of wall shearstress $\overline{\tau}$ for different values of τ_c and u_s against z. It is found that $\overline{\tau}$ decreases up to the zero values of z and then increases. It is also found that $\overline{\tau}$ decreases with the increase of τ_c for fixed values of z but the reverse effect occurs when u_s increases.
The variation of pressure gradient is shown in figure-10. It is observed that the pressure gradient increases up to the zero values of z and then decreases. It is also observed that for fixed values of τ_c and z, the pressure gradient increases with the increase of u_s .

Conclusions:

The present analysis deals with the steady flow of blood through an axially symmetric but radially non-symmetric constricted artery and an axial slip velocity is imposed at the arterial wall. The blood flow characteristics mainly depends on the wall shear stress and pressure gradient. It is clear that thewall shear stress and pressure gradient increase with the increase of u_s . So that slip at the wall of the artery plays a vital role in modelling of blood flow. Thus it may be concluded that with the help of slip velocity, damages of the vessel wall can be repaired.

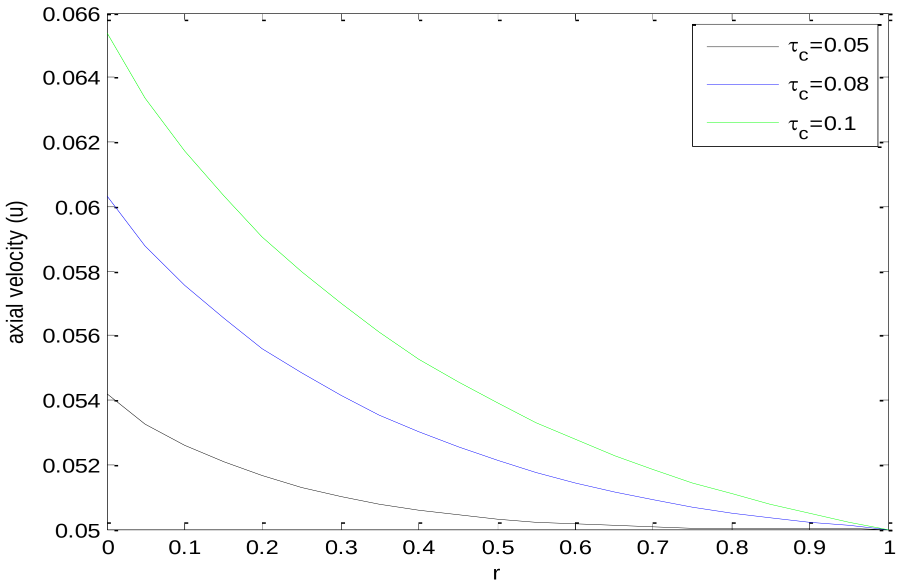

Figure-2

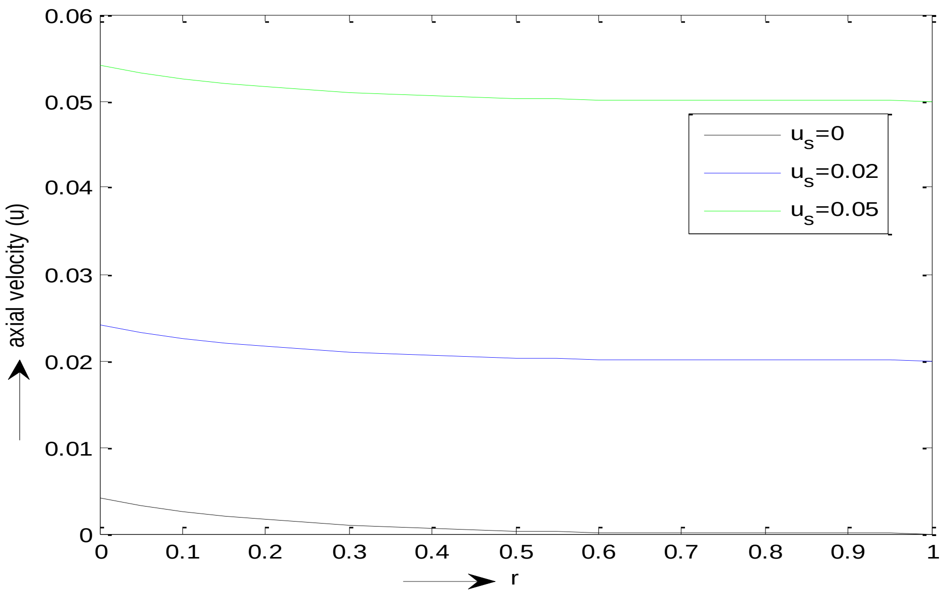

Figure-3

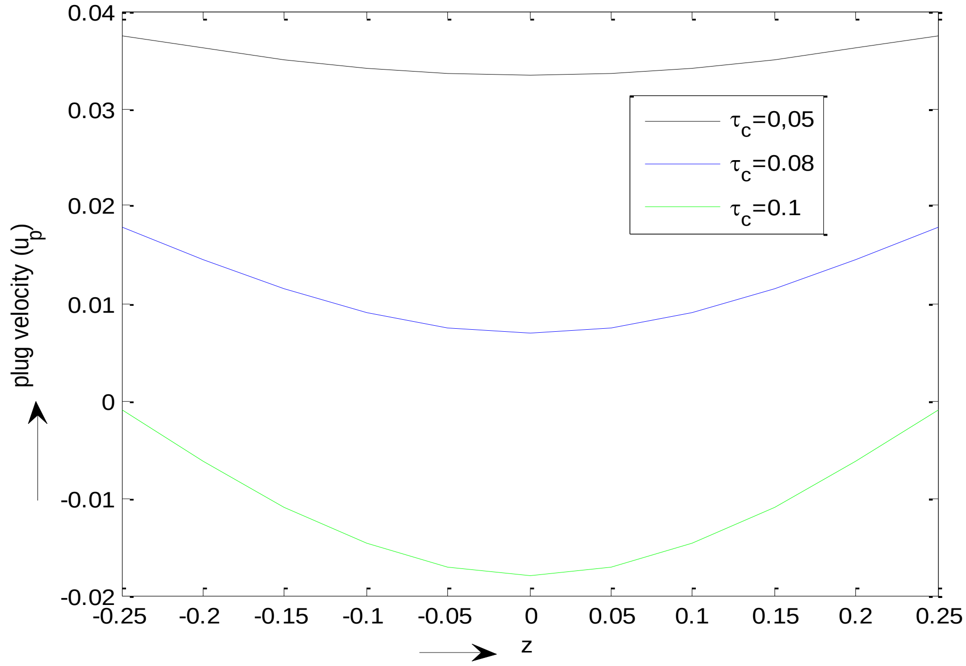

Figure-4

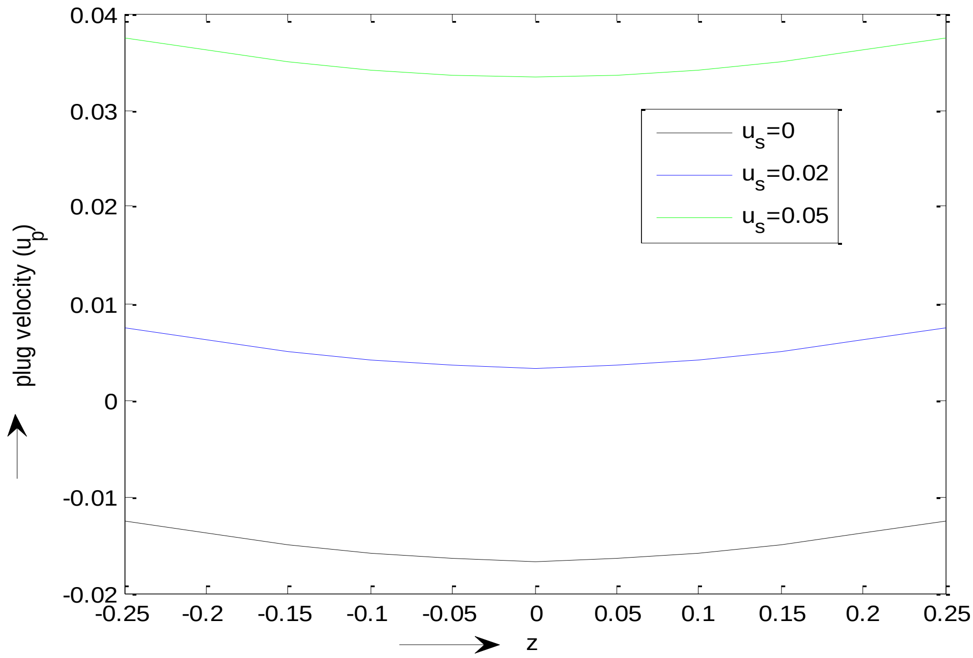

Figure-5

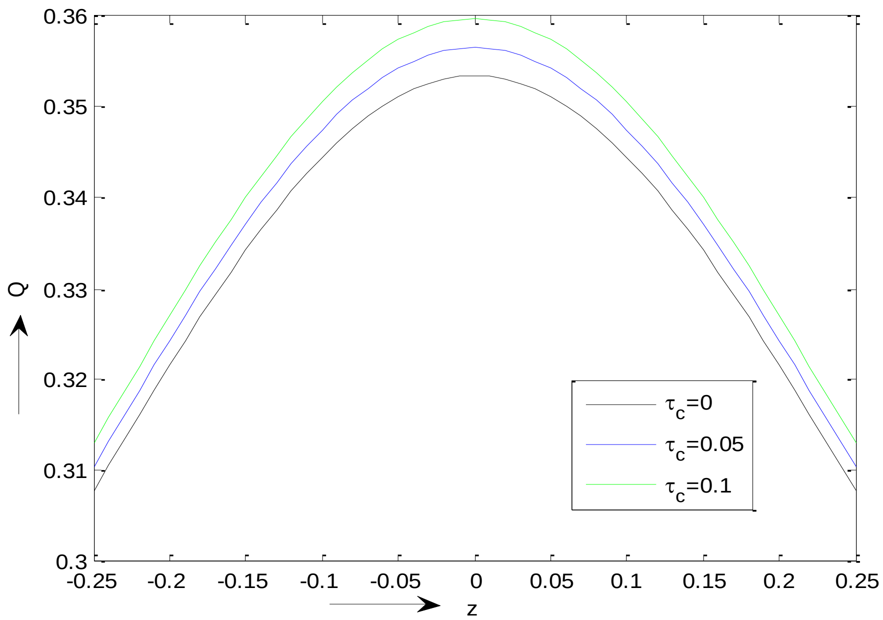

Figure-6

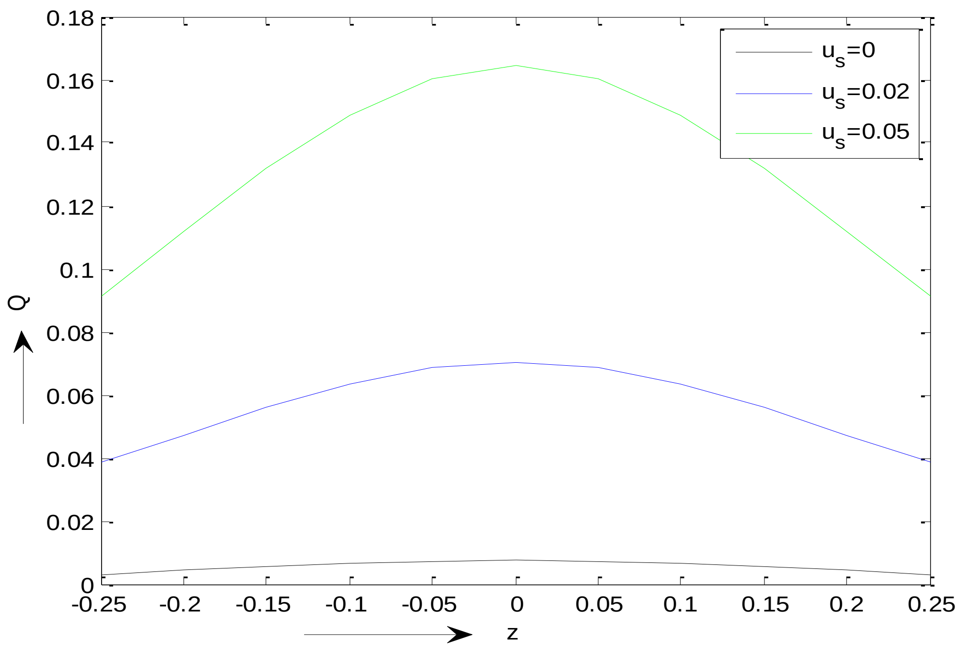

Figure-7

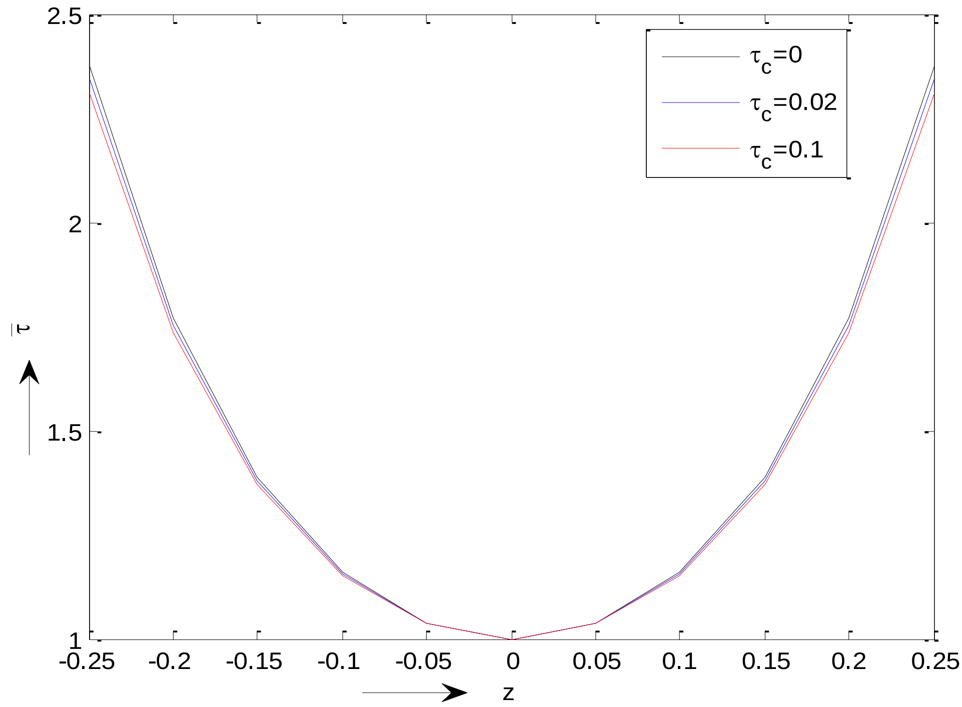

Figure-8

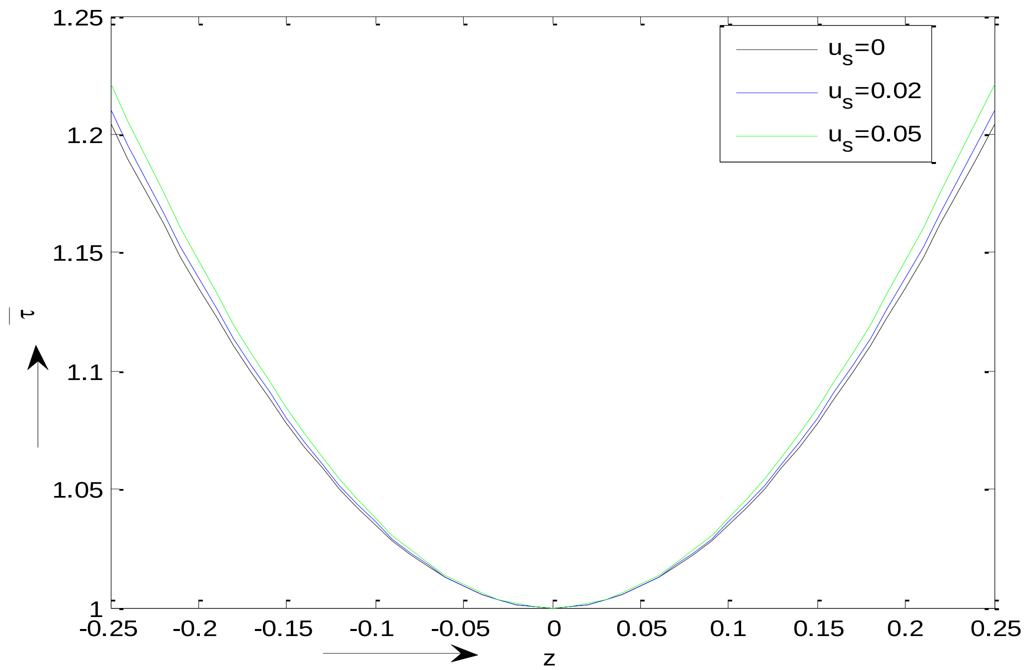

Figure-9

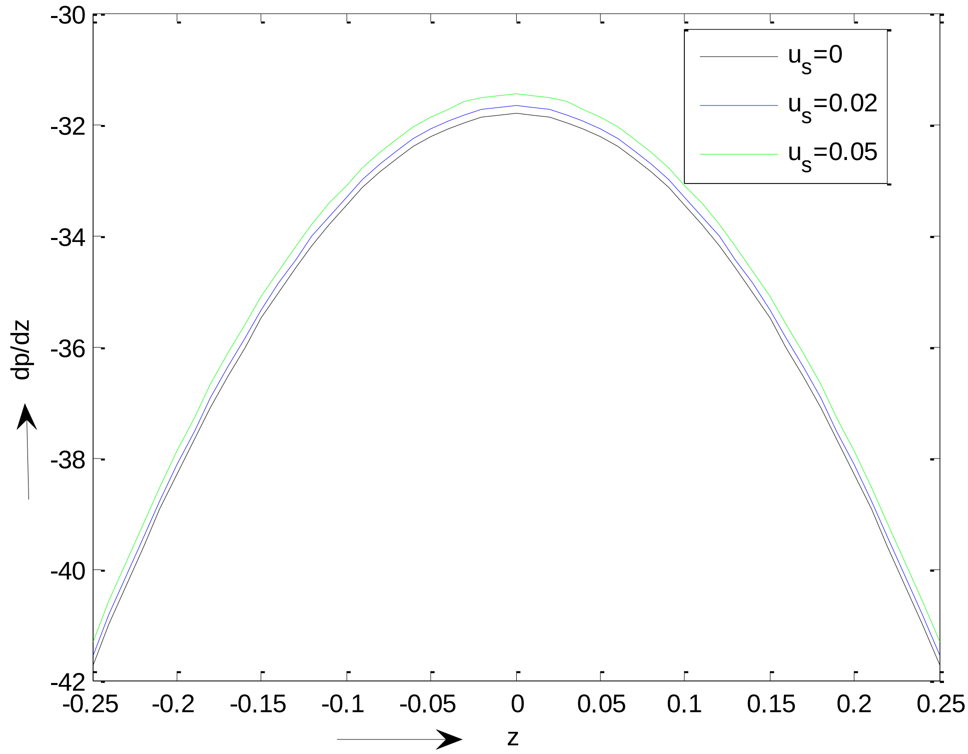

Figure-10

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