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Energy Spectrum in Isotropic Turbulent Flows

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Abstract

The three dimensional energy spectrum function E(k) is related to the spectrum tensor Phi for isotropic turbulence and the one dimensional energy spectrum tensor is function phi of I and j with respect to k 1where phi of one and one with respect to k 1 is the Fourier transform of u squared b of 1 times some function f(r)

Keywords: turbulence, turbulent, isotropic, spectrum, function, tensor, energy, fluid dynamics, flow, instability, unstable, velocity, correlation, condition

The Problem:

The three dimensional energy spectrum function E(k) is related to the spectrum tensor below for isotropic turbulence:

$$\Phi_{ij}(\boldsymbol{k}) = E(k)/4\pi k^2(\delta_{ij} - k_i k_j/k^2)$$

and

$$\int_0^\infty E(k)dk = \frac{3}{2}\bar{u}_1^2$$

The one dimensional energy spectrum tensor is phi of I and j with respect to k 1 such that below is the Fourier transform of u1 bar squared which shows

$$\phi_{11}(k_1) = \int\int dk_2 dk_3 \frac{E(k)}{4\pi k^2}(1 - k_1^2/k^2)$$

And that then

$$E(k) = k^3 \frac{d}{dk}[\frac{1}{k}\frac{d\phi_{11}}{dk}].$$

Proof:

From above, we define the following spectrum tensor function to be the following:

$$\Phi_{ij}(\boldsymbol{k}) = E(k)/4\pi k^2(\delta_{ij} - k_i k_j/k^2)$$

$$E(k) = 4\pi k^2 q(k)$$

Where q(k) is the contributing density in wave number space.

$$\Rightarrow E = \frac{4\pi}{3} tr\delta_i j \int_0^\infty q(k)k^2 dk$$

$$= \int_0^\infty 4\pi k^2 q(k)dk$$

$$= \int_0^\infty E(k)dk = \frac{3}{2}\bar{u}_1^2 = \frac{3}{2}u'^2$$ (isotropy)

However, use 3/2 u one bar squared such that there exists the following:

$$< u_1^2 >= \int_0^\infty E_{11}(\omega)d\omega$$

Where omega is the angular frequency, delta omega is the bandwidth and the numerator below is the filter output:

$$\Rightarrow E_{11}(\omega) = \lim_{\Delta\omega \to 0} \frac{< \Delta u_1^2(\omega) >}{\Delta\omega}$$

Now, we define the following for k one:

$$k_1 = \omega/\bar{U_1}$$

Now, this implies the following:

$$\Rightarrow E_{11}(k_1) = \bar{U_1}E_{11}(\omega)$$

$$\Rightarrow \int_0^{\infty} E_{11}(\omega)d\omega = \int_0^{\infty} \frac{E_{11}(k_1)d\omega}{\bar{U_1}} = \int_0^{\infty} E_{11}(k_1)dk_1 =< u_1^2 >$$

$$\Rightarrow E_{11}(k_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{11}(k_1, k_2, k_3)dk_2dk_3$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} < u_1^2 > f(r_1)cos(k_1, r_1)dr_1$$

$$\Rightarrow E_{11}(k_1) = \int_{k_1}^{\infty} [(1 - \frac{k_1^2}{k^2})\frac{E(k)}{k}]dk$$

$$\Rightarrow E(k) = \frac{k^3 d[k^{-1}dE_{11}(k)/dk]}{dk}$$

$$\therefore E(k) = k^3 \frac{d}{dk}[\frac{1}{k}\frac{d\phi_{11}}{dk}].$$

Compliance with Ethical Standards

The author declares that they have no conflict of interest.

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