Journal of Applied Mathematics and Fluid Mechanics.
ISSN 0974-3170 Volume 10, Number 1 (2018), pp. 31-35

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Incompressible Isotropic Turbulent Flows

Steve Anglin

Sc.M., Ph.D.(h.c.)
Mathematics Dept., Case Western Reserve University, Cleveland, OH, USA

Abstract

For isotropic turbulence, the two-point velocity correlations are given. The condition of incompressible flow then gives a relationship between g(r) and f(r) where 2rg = d(r2f)/dr. Then, we are able to show a special relationship.

Keywords*:* turbulence, turbulent, isotropic, incompressible, fluid dynamics, flow, instability, unstable, velocity, correlation, condition

The Problem:

For isotropic turbulence, the two-point velocity correlations are given.

$$R_{ij}(\boldsymbol{r}) = u_1^2[g(r)\delta_{ij} + [f(r) - g(r)]r_i r_j / r^2]$$

The condition of incompressible flow then gives a relationship between g(r) and f(r) such that we get the following:

$$2rg = \frac{dr^2 f}{dr}$$

Which show these relationships:

$$\lambda_f = \sqrt{2}\lambda_y$$

$$L_f = 2L_g.$$

Proof:

$$R_{ij}(\boldsymbol{r}) = u_1^2[g(r)\delta_{ij} + [f(r) - g(r)]r_i r_j / r^2] \tag{1}$$

$$\frac{\partial R_{ij}}{\partial r_j} = 0 \Rightarrow \frac{\partial u_j}{\partial x_j}(P') = 0$$

Also,

$$u_i(P)\frac{\partial u_j}{\partial x_j}(P') = 0$$

But,

$$x(P') = x(P) + r, \partial x_j(P') = \partial r_j$$

So, that (1) becomes the following:

$$R_{ij} = \frac{f(r) - g(r)}{r^2} r_i r_j + g(r)\delta_{ij}$$

$$r\frac{\partial f(r)}{\partial r} + 2(f(r) - g(r)) = 0$$

$$r\frac{\partial f(r)}{\partial r} + 2f(r) = 2g(r)$$

$$r^2\frac{\partial f(r)}{\partial r} + 2rf(r) = 2rg(r) \qquad (2)$$

$$\frac{\partial}{\partial r}(r^2 f(r)) + 2rf(r) = 2rg(r)$$

$$2rf(r) + r^2\frac{\partial f(r)}{\partial r} + 2rf(r) = 2rg(r)$$

Where f(r) is an even function such that f(r) = f(-r) which implies the following:

$$\Rightarrow 2rf(r) \rightarrow 2rf(r) = 2rf(-r) = -2rf(r)$$

$$\Rightarrow -2rf(r) + r^2\frac{\partial f(r)}{\partial r} + 2rf(r) = 2rg(r)$$

$$\therefore r^2\frac{\partial f(r)}{\partial r} = \frac{\partial}{\partial r}(r^2 f(r)) = 2rg(r) \qquad (3)$$

As r approaches 0, becomes small:

$$r \to 0 \Rightarrow f(r) = 1 - \frac{r^2}{\lambda_f^2} + ..., g(r) = 1 - \frac{r^2}{\lambda_g^2} + ...$$

And, also the following:

$$\frac{\partial^2 f}{\partial r^2}_{r=0} = -2/\lambda_f^2$$

Substitute into equation (3) above, we get the following:

$$\Rightarrow \lambda_f^2 = 2\lambda_g^2 \Rightarrow \lambda_f = \sqrt{2\lambda_g}$$

Now, defining the turbulent integral scales:

$$L_f = \int_0^\infty f(r)dr, L_g = \int_0^\infty g(r)dr$$

From (3) and integration by parts, we show the following:

$$\Rightarrow \frac{L_f}{2} = L_g \Rightarrow L_f = 2L_g.$$

Compliance with Ethical Standards

The author declares that they have no conflict of interest.

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