Image Denoising Based on Curvelet Transforms and its Comparative Study with Basic Filters

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Abstract

Image denoising is basic work for image processing, analysis and computer vision. This Work proposes a Curvelet Transformation based image denoising, which is combined with the low pass filtering and thresholding methods in the transform domain. Through simulations with images contaminated by white Gaussian noise, this scheme exhibits better performance in both PSNR (Peak Signal-to-Noise Ratio) and visual effect as compared to basic filters. Curvelet transformation is a multi-scale transformation technique which is most suitable for the objects with curves.

Introduction

Visual information transmitted in the form of digital images is becoming a major method of communication in the modern age, but the image obtained after transmission is often corrupted with noise. The received image needs processing before it can be used in applications. Image denoising involves the manipulation of the image data to produce a visually high quality image. This thesis reviews the existing denoising algorithms, such as filtering approach, Curvelet approach and performs their comparative study. Different noise models including additive and multiplicative types are used. They include Gaussian noise, salt and pepper noise and speckle noise. Selection of the denoising algorithm is application dependent. Hence, it is necessary to have knowledge about the noise present in the image so as to select the appropriate denoising algorithm. The filtering approach has been proved to be the best when the image is corrupted with salt and pepper noise. The curvelet based approach finds applications in denoising images corrupted with Gaussian noise and speckle noise.
A quantitative measure of comparison is provided by the signal to noise ratio of the image. The Curvelet transform is a higher dimensional generalization of the Wavelet transform designed to represent images at different scales and different angles. Curvelets enjoy two unique mathematical properties:

- Curved singularities can be well approximated with very few coefficients and in a non-adaptive manner.
- Curvelets remain coherent waveforms under the action of the wave equation in a smooth medium. Curvelets are a non-adaptive technique for multi-scale object representation. Being an extension of the wavelet concept, they are becoming popular in similar fields, namely in image processing and scientific computing.

A median filter belongs to the class of nonlinear filters unlike the mean filter. The median filter also follows the moving window principle similar to the mean filter. A $3 \times 3$, $5 \times 5$, or $7 \times 7$ kernel of pixels is scanned over pixel matrix of the entire image. The median of the pixel values in the window is computed, and the center pixel of the window is replaced with the computed median. Median filtering is done by first sorting all the pixel values from the surrounding neighborhood into numerical order and then replacing the pixel being considered with the middle pixel value.

\[
\begin{array}{cccccc}
123 & 125 & 126 & 130 & 140 \\
122 & 124 & 126 & 127 & 135 \\
118 & 120 & 150 & 125 & 134 \\
119 & 115 & 119 & 123 & 133 \\
111 & 116 & 110 & 120 & 130 \\
\end{array}
\]

**Neighborhood values:**
115,119,120,123,124,125,126,127,150

Median value: 124

Wiener Filter is a minimum mean square error filter. It has capabilities of handling both the degradation function as well as the noise, unlike the inverse filter. In Global Wiener a Wiener filter is applied over the whole image. This method does a good job at deblurring; however, it behaves very poorly when the image is corrupted with large noise. The Wiener filter would work well for an image which has similar local statistics throughout the entire image. The advantages of this method are that it is not computationally intensive and that it works well for smooth images (whose local statistics do not vary much from one part of image to another).

A curvelet transform differs from other directional wavelet transforms in that the degree of localisation in orientation varies with scale. In particular, fine-scale basis functions are long ridges; the shape of the basis functions at scale $j$ is $2^{-1}$ by $2^{-1/2}$ so the fine-scale bases are skinny ridges with a precisely determined orientation.

Curvelets are an appropriate basis for representing images (or other functions) which are smooth apart from singularities along smooth curves, where the curves have bounded curvature, i.e. where objects in the image have a minimum length scale. This
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property holds for cartoons, geometrical diagrams, and text. As one zooms in on such images, the edges they contain appear increasingly straight. Curvelets take advantage of this property, by defining the higher resolution curvelets to be skinnier the lower resolution curvelets. However, natural images like photographs do not have this property; they have detail at every scale. Therefore, for natural images, it is preferable to use some sort of directional wavelet transform whose wavelets have the same aspect ratio at every scale.

When the image is of the right type, curvelets provide a representation that is considerably sparser than other wavelet transforms. This can be quantified by considering the best approximation of a geometrical test image that can be represented using only \( n \) wavelets, and analysing the approximation error as a function of \( n \). For a Fourier transform, the error decreases only as \( O(1 / n^{1/2}) \). For a wide variety of wavelet transforms, including both directional and non-directional variants, the error decreases as \( O(1 / n) \). The extra assumption underlying the curvelet transform allows it to achieve \( O((\log(n))^3 / n^2) \).

Efficient numerical algorithms exist for computing the curvelet transform of discrete data. The computational cost of a curvelet transform is approximately 10–20 times that of an FFT, and has the same dependence of \( O(n^2\log(n)) \) for an image of size \( n \times n \).

Digital images have applications in daily life, such as digital cameras, HDTV (High Definition Television) and in areas of research and technology including GIS (Geo-graphical Information System). Datasets collected by image sensors are generally contaminated by noise and noise can be introduced by transmission errors and compression. The purpose of image denoising is to recover an image that is cleaner than its noisy observation. Thus, noise reduction is an important technology in image analysis and the first step to be taken before images are analyzed.

Earlier other methods were used for denoising. Most commonly used denoising methods use low pass filters to get rid of noise. However, both edge and noise information is high-frequency information, so the loss of edge information is evident and inevitable in the denoising process. Edge information is the most important high-frequency information of an image, so we should try to maintain more edge information while denoising. Another image denoising method: wavelet image threshold denoising based on edge detection. Before denoising, those wavelet coefficients of an image that correspond to an image's edges are first detected by wavelet edge detection. The detected wavelet coefficients will then be protected from denoising, and we can therefore set the denoising thresholds based solely on the noise variances, without damaging the image's edges. Finally, we can draw the conclusion that edge detection and denoising are two important branches of image processing. If we combine edge detection with denoising, we can overcome the shortcomings of commonly-used denoising methods and do denoising without notably blurring the edge.
Methodology
Curvelet transform is a multi-scale local transform essentially. First, we used wavelet transform methods, which would decompose signal into a series of different subbands, then using local ridgelet transform for each subband signal. And the size of sub-block in local ridge wave would vary due to scale change. The Curvelet decomposition of function f(x,y) includes the. Shown in Figure 1.

![Diagram of Curvelet Transform]

**Figure 1**: Steps in Curvelet Transform.

**Steps In Curvelet Transform**: The Curvelet decomposition of function f(x,y) includes the following steps:-

1) **Sub-Band Decomposition.** Through wavelet transform the signal was decomposed into multiple sub-band components or layers. Each layer contains details of different frequencies:
   a) $P_0$ – Low-pass filter.
   b) $\Delta_1, \Delta_2, \ldots$ – Band-pass (high-pass) filters.

   The original image can be reconstructed from the sub-bands:
   - $P_0$ is “smooth” (low-pass), and can be efficiently represented using wavelet Base.
   - The discontinuity curves effect the high-pass layers $\Delta s f$.

   Can they be represented efficiently?
   - Looking at a small fragment of the curve, it appears as a relatively straight ridge.
   - We will dissect the layer into small partitions.

2) **Smooth partitioning.** In this step, the high frequency band will be divided into several sub-blocks, and different subband components can be divided into different sub-block size.

3) **Ridgelet Decomposition.** The Ridgelet transform was used to each sub-block which was from smoothing & segmenting sub-band.
Results
The curvelet technique as well as other techniques is applied on sample images. The table 4.1 shows results of proposed technique.

Table 4.1: Results.

<table>
<thead>
<tr>
<th>Type of Noise</th>
<th>Technique Used</th>
<th>PSNR Value of Input Image</th>
<th>PSNR Value of output image</th>
</tr>
</thead>
<tbody>
<tr>
<td>SALT &amp; PEPPER</td>
<td>MEDIAN</td>
<td>26.8126</td>
<td>26.9528</td>
</tr>
<tr>
<td></td>
<td>WEINER</td>
<td>26.8126</td>
<td>21.7797</td>
</tr>
<tr>
<td></td>
<td>CURVELET</td>
<td>26.8126</td>
<td>18.4485</td>
</tr>
<tr>
<td>GUASIAN</td>
<td>MEDIAN</td>
<td>15.4422</td>
<td>18.1607</td>
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<tr>
<td></td>
<td>WEINER</td>
<td>15.4422</td>
<td>18.5616</td>
</tr>
<tr>
<td></td>
<td>CURVELET</td>
<td>15.4422</td>
<td>18.7537</td>
</tr>
<tr>
<td>SPECKLE</td>
<td>MEDIAN</td>
<td>15.1162</td>
<td>17.1347</td>
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<tr>
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<td>WEINER</td>
<td>15.1162</td>
<td>17.6553</td>
</tr>
<tr>
<td></td>
<td>CURVELET</td>
<td>15.1162</td>
<td>18.7570</td>
</tr>
</tbody>
</table>

In case of salt & pepper noise median filter is best but for other types of noises Guasian & Speckle noise curvelet technique is best. The median filter provides a solution to this, where the sharpness of the image is retained after denoising. From our experimentation, it has been observed that the filtering approach does not produce considerable denoising for images corrupted with Gaussian noise or speckle noise. Curvelet play a very important role in the removal of the noise, especially where curves are involved.

Curvelet Transform give a superior performance in image denoising due to properties such as sparsity and multiresolution structure. Through simulations with images contaminated by white Gaussian noise, this scheme exhibits better performance in both PSNR (Peak Signal-to-Noise Ratio) and visual effect.

Conclusion
The Curves in case of normal transformation techniques like FFT, DWT and DCT are considered as edges which are further treated as object boundaries. These boundary conditions can affect the transformation results both in time and frequency domains. So curvelet is a technique which uses Wavelet transformation as the core but without skipping the curves. We developed a simple and efficient algorithm based on the Curvelet Transform threshold for image denoising, which combines Low pass and High pass filter depending upon the relevant Frequency of the image with respect to threshold value. Curvelet Transform has efficient noise reduction ability; wavelets still have problems on a heavy noisy network. We investigate the problem of image denoising when the source image is corrupted by additive white Gaussian noise, which
is a valid assumption for images obtained through transmitting, scanning or compression. From the experimental and mathematical results it can be concluded that for salt and pepper noise, the median filter is optimal compared to weiner filter and curvelet transform. It produces the maximum SNR for the output image compared to the other filters considered. From the output images shown, the image obtained from the median filter has no noise present in it and is close to the high quality image. The sharpness of the image is retained unlike in the case of other filtering techniques. In the case where an image is corrupted with Gaussian noise & speckle noise, the curvelet denoising has proved to be nearly optimal. It produces the best PSNR compared to other filters. However, the output from Curvelet method is much closer to the high quality image and there is no blurring in the output image unlike the other two methods. Curvelets are an appropriate basis for representing images (or other functions) which are smooth apart from singularities along smooth curves, where the curves have bounded curvature, i.e. where objects in the image have a minimum length scale.

References


